International Journal of Science, Environment and
Technology, Vol. 1, No 1, 7-18, 2012

THE POINT SPECTURM FOR χ^2 HOUSDORFF MATRICES

N. Subramanian
Department of Mathematics, SASTRA University,
Thanjavur-613 401, India.
E-mail: nsmaths@yahoo.com

Abstract: In a recent paper Okutayi and Thrope [54] determined the spectrum of the double Ces`aro matrix of order 1. In this paper we determine the point spectrum of a wide class of double Housdorff matrices, considered as operators over $\chi^2(\chi^2)$.

Keywords: Double gai sequences, Double analytic, Housdorff Matrices.
2000 Mathematics subject classification: 40A05, 40C05, 40D05.

1 Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences respectively. We write w^2 for the set of all complex sequences (x_{mn}), where $m, n \in N$, the set of positive integers. Then w^2 is a linear space under the coordinate wise addition and scalar multiplication.

Some initial works on double sequence spaces is found in Bromwich [5]. Later on, they were investigated by Hardy [15], Moricz [26], Moricz and Rhoades [27], Basarir and Solancan [3], Tripathy [47], Colak and Turkmenoglu [8], Turkmenoglu [49], and many others. Let us define the following sets of double sequences:

$$\mathcal{M}(t) := \{(x_{mn}) \in w^2 : \sup_{m,n \in N} |x_{mn}|^{t_{mn}} < \infty\},$$

$$\mathbb{C}(t) := \{(x_{mn}) \in w^2 : p - \lim_{m,n \to \infty} |x_{mn} - l|^{t_{mn}} = 1, \text{ for some } l \in \mathrm{C}\},$$

$$\mathcal{C}_{0p}(t) := \left\{ (x_{mn}) \in w^2 : p - lim_{m,n\to\infty} |x_{mn}|^{t_{mn}} = 0 \right\},$$
$$\mathcal{L}_u(t) := \left\{ (x_{mn}) \in w^2 : \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} |x_{mn}|^{t_{mn}} < \infty \right\},$$
$$\mathcal{C}_{bp}(t) := \mathcal{C}_p(t) \bigcap \mathcal{M}_u(t) \text{ and } \mathcal{C}_{0bp}(t) = \mathcal{C}_{0p}(t) \bigcap \mathcal{M}_u(t);$$

where $t = (t_{mn})$ is the sequence of strictly positive reals t_{mn} for all $m, n \in \mathbb{N}$ and $p - lim_{m,n\to\infty}$ denotes the limit in the Pringsheim's sense. In the case $t_{mn} = 1$ for all $m, n \in \mathbb{N}; \mathcal{M}_u(t), \mathcal{C}_p(t), \mathcal{C}_{0p}(t), \mathcal{L}_u(t), \mathcal{C}_{bp}(t)$ and $\mathcal{C}_{0bp}(t)$ reduce to the sets $\mathcal{M}_u, \mathcal{C}_p, \mathcal{C}_{0p}, \mathcal{L}_u, \mathcal{C}_{bp}$ and $\mathcal{C}_{0bp}$, respectively. Now, we may summarize the knowledge given in some document related to the double sequence spaces. Gökhan and Colak [13,14] have proved that $\mathcal{M}_u(t)$ and $\mathcal{C}_p(t), \mathcal{C}_{bp}(t)$ are complete paranormed spaces of double sequences and gave the $\alpha-, \beta-, \gamma-$ duals of the spaces $\mathcal{M}_u(t)$ and $\mathcal{C}_{bp}(t)$. Quite recently, in her PhD thesis, Zelter [52] has essentially studied both the theory of topological double sequence spaces and the theory of summability of double sequences. Mursaleen and Edely [29] have recently introduced the statistical convergence and Cauchy for double sequences and given the relation between statistical convergent and strongly Cesàro summable double sequences. Nextly, Mursaleen [30] and Mursaleen and Edely [31] have defined the almost strong regularity of matrices for double sequences and applied these matrices to establish a core theorem and introduced the $M-$core for double sequences and determined those four dimensional matrices transforming every bounded double sequences $x = (x_{jk})$ into one whose core is a subset of the $M-$core of x. More recently, Altay and Basar [2] have defined the spaces $\mathcal{BS}, \mathcal{BS}(t), \mathcal{CS}_p, \mathcal{CS}_{bp}, \mathcal{CS}_r$ and $\mathcal{BV}$ of double sequences consisting of all double series whose sequence of partial sums are in the spaces $\mathcal{M}_u, \mathcal{M}_u(t), \mathcal{C}_p, \mathcal{C}_{bp}, \mathcal{C}_r$ and $\mathcal{L}_u$, respectively, and also examined some properties of those sequence spaces and determined the $\alpha-$ duals of the spaces $\mathcal{BS}, \mathcal{BV}, \mathcal{CS}_{bp}$ and the $\beta(\vartheta)-$ duals of the spaces $\mathcal{CS}_{bp}$ and $\mathcal{CS}_r$ of double series. Quite recently Basar and Sever [7] have introduced the Banach space $\mathcal{L}_q$ of double sequences corresponding to the well-known space ℓ_q of single sequences and examined some properties of the space $\mathcal{L}_q$. Quite recently Subramanian and Misra [40] have studied the space $\chi^2_M(p, q, u)$ of double sequences and gave some inclusion relations.

Spaces are strongly summable sequences were discussed by Kuttner [23], Maddox [32], and others. The class of sequences which are strongly Cesàro summable with respect to a modulus was introduced by Maddox [25] as an extension of the definition of strongly Cesàro summable sequences. Connor [10] further extended this definition to a definition of strong $A-$ summability with respect to a modulus where $A = (a_{n,k})$ is a nonnegative regular matrix and established some connections between strong $A-$ summability, strong $A-$ summability with respect to a modulus, and $A-$ statistical convergence.

In [36] the notion of convergence of double sequences was presented by A. Pringsheim. Also, in [17]-[19], and [38] the four dimensional matrix transformation $(Ax)_{k,\ell} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{k\ell}^{mn} x_{mn}$ was studied extensively by Robison and Hamilton.
Let $p = (p_{mn})$ be a sequence of positive real numbers with $0 < p_{mn} < supp_{mn} = G$ and let $D = max\left(1, 2^{G-1}\right)$. Then for $a_{mn}, b_{mn} \in \mathbb{C}$, the set of complex numbers for all $m, n \in \mathbb{N}$ we have

$$|a_{mn} + b_{mn}|^{\frac{1}{m+n}} \leq D \left\{ |a_{mn}|^{\frac{1}{m+n}} + |b_{mn}|^{\frac{1}{m+n}} \right\} \tag{1}$$

We need the following inequality in the sequel of the paper. For $a, b, \geq 0$ and $0 < p < 1$, we have

$$(a + b)^p \leq a^p + b^p \tag{2}$$

An FK-space(or a metric space)X is said to have AK property if $(\Im_{mn})$ is a Schauder basis for X. Or equivalently $x^{[m,n]} \to x$.
If X is a sequence space, we give the following definitions:

(i)X'= the continuous dual of X;

(ii)$X^{\alpha} = \left\{ a = (a_{mn}) : \sum_{m,n=1}^{\infty} |a_{mn} x_{mn}| < \infty, \, for \;\; each \;\; x \in X \right\};$

(iii)$X^{\beta} = \left\{ a = (a_{mn}) : \sum_{m,n=1}^{\infty} a_{mn} x_{mn} \;\; is \;\; convergent, \, for \; each \; x \in X \right\};$

(iv)$X^{\gamma} = \left\{ a = (a_{mn}) : sup_{mn} \geq 1 \left| \sum_{m,n=1}^{M,N} a_{mn} x_{mn} \right| < \infty, \;\; for \;\; each \;\; x \in X \right\};$

(v)$Let \;\; X \;\; be \; an \; FK - space \supset \phi; \, then \;\; X^f = \left\{ f(\Im_{mn}) : f \in X' \right\};$

(vi)$X^{\delta} = \left\{ a = (a_{mn}) : sup_{mn} |a_{mn} x_{mn}|^{1/m+n} < \infty, \, for \;\; each \;\; x \in X \right\};$

$X^{\alpha}.X^{\beta}, X^{\gamma}$ are called $\alpha-(or K\ddot{o}the-Toeplitz)$dual of $X, \beta-(or\, generalized- K\ddot{o}the - Toeplitz)\, dual\, of\, X, \gamma -\, dual\, of\, X,\, \delta -\, dual\, of X\, respectively. X^{\alpha}$ is defined by Gupta and Kamptan [12]. It is clear that $x^{\alpha} \subset X^{\beta}$ and $X^{\alpha} \subset X^{\gamma}$, but $X^{\alpha} \subset X^{\gamma}$ does not hold, since the sequence of partial sums of a double convergent series need not to be bounded.

The notion of difference sequence spaces (for single sequences) was introduced by Kizmaz [22] as follows

$$Z(\Delta) = \{x = (x_k) \in w : (\Delta x_k) \in Z\}$$

for $Z = c, c_0$ and ℓ_∞, where $\Delta x_k = x_k - x_{k+1}$ for all $k \in \mathbb{N}$. Here w, c, c_0 and ℓ_∞ denote the classes of all, convergent,null and bounded sclar valued single sequences respectively. The above spaces are Banach spaces normed by

$$\|x\| = |x_1| + sup_{k\geq 1} |\Delta x_k|$$

Later on the notion was further investigated by many others. We now introduce the following difference double sequence spaces defined by

$$Z(\Delta) = \{x = (x_{mn}) \in w^2 : (\Delta x_{mn}) \in Z\}$$

where $Z = \Lambda^2, \chi^2$ and $\Delta x_{mn} = (x_{mn} - x_{mn+1}) - (x_{m+1n} - x_{m+1n+1}) = x_{mn} - x_{mn+1} - x_{m+1n} + x_{m+1n+1}$ for all $m, n \in \mathbb{N}$
$\Delta^m x_{mn} = \Delta\Delta^{m-1} x_{mn}$ for all $m, n \in \mathbb{N}$, where
$\Delta^m x_{mn} = \Delta^{m-1} x_{mn} - \Delta^{m-1} x_{mn+1} - \Delta^{m-1} x_{m+1,n} + \Delta^{m-1} x_{m+1,n+1}$, for all $m, n \in \mathbb{N}$.
The double series $\sum_{m,n=1}^{\infty} x_{mn}$ is called convergent if and only if the double sequence (s_{mn}) is convergent, where $s_{mn} = \sum_{i,j=1}^{m,n} x_{ij} (m, n \in \mathbb{N})$ (see[1]); which will be used throughout. For the sake of brevity, here and in what follows, we abbreviate the summations $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty}$; $\sum_{i=1}^{m} \sum_{j=1}^{n}$ and $\sum_{i=1}^{n} \sum_{j=1}^{n}$ by $\sum_{ij}$; $\sum_{i,j=1}^{m,n}$ and $\sum_{i,j=1}^{n}$ respectively. Then the pair (x, s) and the sequence $s = (s_{mn})$ are called as a double series and the sequence of partial sums of the double series respectively. Let X be the space of double sequences, converging with respect to some linear convergence rule $v - lim : X \to \Re$. The sum of a double series $\sum_{i,j} x_{ij}$ with respect to this rule is defined by $v - \sum_{i,j} x_{ij} := v - lim s_{mn}$. We denote w^2 and Ω are called as the double sequence spaces respectively.

A double sequence $x = (x_{mn})$ is called convergent (with limit L) if and only if for every $\epsilon > 0$ there exists a positive integer $n_0 = n_0(\epsilon)$ such that $|x_{mn} - L| < \epsilon$, for all $m, n \geq n_0$. We write $x_{mn} \to L$ or $lim_{m,n\to\infty} x_{mn} = L$ if (x_{mn}) is convergent to L. The limit L is called double limit or Pringsheim sense limit.

In the present paper, we introduce the space χ^2 :

A sequence $x = (x_{mn}) \in w^2$ is called a double gai sequence if

$$((m+n)! |x_{mn}|)^{1/m+n} \to 0 \text{ as } m, n \to \infty.$$

We denote χ^2 as the class of prime sense double gai sequences. The space χ^2 is a metric space with the metric

$$\tilde{d}(x, y) = sup_{mn} \left\{ ((m+n)! |x_{mn} - y_{mn}|)^{1/m+n} : m, n = 1, 2, \cdots \right\} \quad (3)$$

for all $x = (x_{mn})$ and $y = (y_{mn})$ in χ^2, respectively.

Consider a double sequence $x = (x_{ij})$. The $(m,n)^{th}$ section $x^{[m,n]}$ of the sequence is defined by $x^{[m,n]} = \sum_{i,j=0}^{m,n} x_{ij}\Im_{ij}$ for all $m, n \in \mathbb{N}$; where $\Im_{ij}$ denotes the double sequence whose only non zero term is $\frac{1}{(i+j)!}$ in the $(i,j)^{th}$ place for each $i, j \in \mathbb{N}$.

A double Housdorff matrix is a doubly infinite matrix with nonzero entires

$$h_{mn}^{ij} = \binom{m}{i}\binom{n}{j}\sum_{r=0}^{m-i}\sum_{s=0}^{n-j}(-1)^{r+s}\binom{m-i}{r}\binom{n-j}{s}\mu_{i+r,j+s}, 0 \le i \le m, 0 \le j \tag{4}$$

$\le n$, where $\{\mu_{ij}\}$ is a complex sequence.

We shall let H_μ denote the corresponding Housdorff matrix, and $\sigma_q(H_\mu)$ denote the point spectrum of H_μ.

2. Lemma

For $m, n, = 0, 1, \cdots, \sum_{i=0}^{m}\sum_{j=0}^{n} h_{mn}^{ij} = \mu_{00}$.

3. Main Results

Theorem 3.1.

Let H_μ be a bounded operator on $\chi^2(\chi^2)$, with the $\{\mu_{mn}\}$ distinct and nonzero then $\sigma_q(H_\mu) = \phi$

Proof: Define $\Delta_{10}a_{ij} = a_{ij} - a_{i+1,j}, \quad \Delta_{01}a_{ij} = a_{ij} - a_{i,j+1},$

$$E_{10}a_{ij} = a_{i+1,j}, \quad E_{01}a_{ij} = a_{i,j+1}. \text{ With}$$

$$t_{mn} = \sum_{i=0}^{m}\sum_{j=0}^{n} h_{mn}^{ij}\left((i+j)!\,|x_{ij}|\right)^{1/i+j}.$$

we are interested in solving the system

$$t_{mn} = \lambda\left((m+n)!\,|x_{mn}|\right)^{1/m+n}. \tag{5}$$

Suppose that x is an eigen vector of (5) with $\left((0!)\,|x_{00}|\right)^{1/0} \neq 0$. From (5) with $m = n = 0$ it follows that $\lambda = \mu_{00}$.

Using (5) with $m = 1, n = 0$ implies that $((1!)\,|x_{10}|)^{1/1} = ((0!)\,|x_{00}|)^{1/0}$, since the μ_{jn} are all distinct.

Claim 1. $((p!)\,|x_{p0}|)^{1/p} = ((0!)\,|x_{00}|)^{1/0} \quad for \quad p = 1, 2, \cdots$.

The claim has already been verified for $p = 1$. Assume the induction hypothesis and consider

$\mu_{00}\,((p+1)!\,|x_{p+10}|)^{1/p+1} = t_{p+10} = \sum_{i=0}^{p+1} h_{p+1,0}^{i,0}\,((i!)\,|x_{i0}|)^{1/i}$
$h_{m+1,0}^{p+1,0}\,(((p+1)!)\,|x_{p+10}|)^{1/p+1}$.
$= \sum_{i=0}^{p} h_{p+1,0}^{i,0}\,((i!)\,|x_{i0}|)^{1/i} +$

Thus

$(\mu_{00} - \mu_{p+1,0})\,(((p+1)!)\,|x_{p+10}|)^{1/p+1} = ((0!)\,|x_{00}|)^{1/0} \sum_{i=1}^{p} h_{p+1,0}^{i,0} =$
$((0!)\,|x_{00}|)^{1/0} \left(\sum_{i=0}^{p+1} h_{p+1,0}^{i,0} - h_{p+1,0}^{p+1,0}\right)$.

Using the Lemma we obtain $(\mu_{00} - \mu_{m+1,0}) = ((0!)\,|x_{00}|)^{1/0}\,(\mu_{00} - \mu_{p+1,0})$, which implies that $(((p+1)!)\,|x_{p+10}|)^{1/p+1} = ((0!)\,|x_{00}|)^{1/0} = 0$, since the μ_{jk} are distinct. Therefore $\{x_{mn}\} \notin \chi^2\,(\chi^2)$ and $((0!)\,|x_{00}|)^{1/0} = 0$.

Either $((m!)\,|x_{0m}|)^{1/m} = 0$ for all m, Or there exists a smallest positive integer N such that $((N!)\,|x_{0N}|)^{1/N} \neq 0$.

Assume the latter condition. Then, from (5), with $m = 0, n = N$, it follows that $\lambda = \mu_{0N}$.

Claim 2: $(((N+m)!)\,|x_{0N+m}|)^{1/N+m} = \binom{N+m}{m}\,((N!)\,|x_{0N}|)^{1/N}$ for $m = 0, 1, 2 \cdots$.

The claim is trivially true for $m = 0$. Assume the induction hypothesis. From (5), with $m = 0, n = N + m + 1$,

$\lambda\,(((N+m+1)!)\,|x_{0N+m+1}|)^{1/N+m+1} = t_{0,N+m+1}$

$= \sum_{j=0}^{N+m+1} h_{0,N+m+1}^{0,j}\,((j!)\,|x_{0j}|)^{1/j} =$

$\sum_{j=N}^{N+m+1} h_{0,N+m+1}^{0,j}\,((j!)\,|x_{0j}|)^{1/j} = \sum_{j=N}^{N+m} h_{0,N+m+1}^{0,j}\,((j!)\,|x_{0j}|)^{1/j} +$
$h_{0,N+m+1}^{0,N+m+1}\,(((N+m+1)!)\,|x_{0N+m+1j}|)^{1/N+m+1}$.

Thus

$(\mu_{0N} - \mu_{0,N+m+1})\,(((N+m+1)!)\,|x_{0N+m+1}|)^{1/N+m+1}$

$= \sum_{i=0}^{m} h_{0,N+m+1}^{0,i+N}\,(((i+N)!)\,|x_{0i+N}|)^{1/i+N}$

$= \sum_{i=0}^{m} \binom{N+m+1}{i+N} \sum_{S=0}^{m+1-i} (-1)^S \binom{m+1-i}{S} \mu_{0,i+N+S} \binom{N+i}{i} \left((N!)\,|x_{0N}|\right)^{1/N}.$

Since

$$\binom{N+m+1}{i+N}\binom{N+i}{i} = \binom{N+m+1}{m+1}\binom{m+1}{i},$$

the right hand side of the equation becomes

$$\left((N!)\,|x_{0N}|\right)^{1/N} \binom{N+m+1}{m+1} \sum_{i=0}^{m} \binom{m+1}{i} \sum_{S=0}^{m+1-i} (-1)^s \binom{N+1-i}{S} \mu_{0,i+N+S} =$$
$$\left((N!)\,|x_{0N}|\right)^{1/N} \binom{N+m+1}{m+1} \left[\sum_{i=0}^{m+1} \binom{m+1}{i} \Delta_{01}^{m+1-i} E_{01}^{i} \mu_{0N} - \mu_{0N+M+1}\right] =$$
$$\left((N!)\,|x_{0N}|\right)^{1/N} \binom{N+m+1}{m+1} \left(\mu_{0N} - \mu_{0,N+m+1}\right),$$

since the μ_{jk} are distinct. Therefore $\{x_{mn}\} \notin \chi^2(\chi^2)$ and $\left((m!)\,|x_{0m}|\right)^{1/m} = 0$ for all m.

Similarly it can be shown that, if there exists a smallest postive integer M such that $\left((M!)\,|x_{0M}|\right)^{1/M} \neq 0$ then $\{x_{mn}\} \notin \chi^2(\chi^2)$. Therefore $\left((p!)\,|x_{p0}|\right)^{1/p} = 0$ for all p

To complete the proof that $\left(((m+n)!)\,|x_{mn}|\right)^{1/m+n} = 0$ for $m, n = 0, 1, 2, \cdots$, we shall assume as an induction hypothesis that $\left(((i+n)!)\,|x_{in}|\right)^{1/i+n} = 0$ for $0 \leq i \leq m$ and all n, and that $\left(((m+j)!)\,|x_{mj}|\right)^{1/m+j} = 0$ for $0 \leq j \leq n$ for all m.

We shall first show that $\left(((m+n+2)!)\,|x_{m+1n+1}|\right)^{1/m+n+2} = 0$.

Suppose not. Then, from (5) with $m = m+1, n = n+1$,

$$\lambda \left(((m+n+2)!)\,|x_{m+1n+1}|\right)^{1/m+n+2} = t_{m+1n+1} = \sum_{i=0}^{m+1} \sum_{j=0}^{n+1} h_{m+1,n+1}^{i,j} \left(((i+j)!)\,|x_{ij}|\right)^{1/i+j} = h_{m+1,n+1}^{m+1,n+1} \left(((m+n+2)!)\,|x_{m+1n+1}|\right)^{1/m+n+2},$$

which implies from (4), that $\lambda = \mu_{m+1,n+1}$.

By induction it can be shown that

$$\left(((m+n+p+2)!)\,|x_{m+1+p,n+1}|\right)^{1/m+n+p+2} = \binom{m+1+p}{p} \left(((m+n+2)!)\,|x_{m+1n+1}|\right)^{1/m+n+2}$$

and $\{x_{mn}\} \notin \chi^2(\chi^2)$. Therefore $\left(((m+n+2)!)\,|x_{m+1n+1}|\right)^{1/m+n+2} = 0$.

We shall now show that $\left(((q+n+1)!)\left|x_{qn+1}\right|\right)^{1/q+n+1}=0$ for all $q\geq m$ and that $\left(((m+r+1)!)\left|x_{m+1r}\right|\right)^{1/m+r+1}=0$ for $r\geq n$.

Suppose that there exists a smallest integer $M>m+1$ such that $\left(((M+n+2)!)\left|x_{M+1n+1}\right|\right)^{1/M+n+2}\neq 0$. From (5),

$$\lambda\left(((M+n+2)!)\left|x_{M+1n+1}\right|\right)^{1/M+n+2}=t_{M+1n+1}$$

$$=\sum_{i=0}^{M+1}\sum_{j=0}^{n+1}h_{M+1,n+1}^{i,j}\left(((i+j)!)\left|x_{ij}\right|\right)^{1/i+j}=$$

$$\sum_{j=0}^{n+1}h_{M+1,n+1}^{M+1,j}\left(((M+j+1)!)\left|x_{M+1j}\right|\right)^{1/M+j+1}=$$
$$h_{M+1,n+1}^{M+1,n+1}\left(((M+n+2)!)\left|x_{M+1n+1}\right|\right)^{1/M+n+2},$$

which implies that $\lambda=\mu_{M+1n+1}$.

By induction it can be shown that

$$\left(((M+p+n+2)!)\left|x_{M+p+1n+1}\right|\right)^{1/M+p+n+2}$$
$$=\binom{M+p+1}{p}\left(((M+n+2)!)\left|x_{M+1n+1}\right|\right)^{1/M+n+2}$$

and $\{x_{mn}\}\notin\chi^2\left(\chi^2\right)$. Therefore $\left(((m+n+2)!)\left|x_{m+1n+1}\right|\right)^{1/m+n+2}=0$ for all m.

Similarly, it can be shown that $\left(((m+n+2)!)\left|x_{m+1n+1}\right|\right)^{1/m+n+2}=0$ for all n, and $\sigma_q\left(H^{\mu}\right)=\phi$.

3.2 Corollary

Theorem 1 of [54] is the special case of above theorem with $\chi^2\left(\chi^2\right)$ and $\mu_{mn}=\frac{1}{(m+1)\times(n+1)}$.

References

[1] T.Apostol, Mathematical Analysis, *Addison-wesley , London*, **1**978.

[2] B.Altay and F.Basar, Some new spaces of double sequences, *J. Math. Anal. Appl.*, **309(1)**, (2005), 70-90.

[3] M.Basarir and O.Solancan, On some double sequence spaces, *J. Indian Acad. Math.*, **21(2)** (1999), 193-200.

[4] C.Bektas and Y.Altin, The sequence space $\ell_M(p,q,s)$ on seminormed spaces, *Indian J. Pure Appl. Math.*, **34(4)** (2003), 529-534.

[5] T.J.I'A.Bromwich, An introduction to the theory of infinite series *Macmillan and Co.Ltd.* ,New York, (1965).

[6] J.C.Burkill and H.Burkill, A Second Course in Mathematical Analysis *Cambridge University Press, Cambridge*, New York, (1980).

[7] F.Basar and Y.Sever, The space $\mathcal{L}_p$ of double sequences, *Math. J. Okayama Univ*, **51**, (2009), 149-157.

[8] R.Colak and A.Turkmenoglu, The double sequence spaces $\ell^2_\infty(p), c^2_0(p)$ and $c^2(p)$, *(to appear)*.

[9] K.Chandrasekhara Rao and N.Subramanian, The Orlicz space of entire sequences, *Int. J. Math. Math. Sci.*, **68**(2004), 3755-3764.

[10] J.Cannor, On strong matrix summability with respect to a modulus and statistical convergence, *Canad. math. Bull*, **32(2)**, (1989), 194-198.

[11] M.Gupta and P.K.Kamthan,Infinite matrices and tensorial transformations, *Acta Math.* , **Vietnam 5** (1980), 33-42.

[12] M.Gupta and P.K.Kamthan, Infinite Matrices and tensorial transformations, *Acta Math. Vietnam* **5**, (1980), 33-42.

[13] A.Gökhan and R.Colak, The double sequence spaces $c^P_2(p)$ and $c^{PB}_2(p)$, *Appl. Math. Comput.*, **157(2)**, (2004), 491-501.

[14] A.Gökhan and R.Colak, Double sequence spaces ℓ^∞_2, *ibid.*, **160(1)**, (2005), 147-153.

[15] G.H.Hardy, On the convergence of certain multiple series, *Proc. Camb. Phil. Soc.*, **19** (1917), 86-95.

[16] H.J.Hamilton, Transformations of multiple sequences, *Duke Math, Jour.*, **2**, (1936), 29-60.

[17] ———-, A Generalization of multiple sequences transformation, *Duke Math, Jour.*, **4**, (1938), 343-358.

[18] ———-, Change of Dimension in sequence transformation , *Duke Math, Jour.*, **4**, (1938), 341-342.

[19] ———-, Preservation of partial Limits in Multiple sequence transformations, *Duke Math, Jour.*, **4**, (1939), 293-297.

[20] M.A.Krasnoselskii and Y.B.Rutickii, Convex functions and Orlicz spaces, *Gorningen, Netherlands*, **1961**.

[21] P.K.Kamthan and M.Gupta, Sequence spaces and series, Lecture notes, Pure and Applied Mathematics, *65 Marcel Dekker, In c., New York* , **1981**.

[22] H.Kizmaz, On certain sequence spaces, *Cand. Math. Bull.*, **24(2)**, (1981), 169-176.

[23] B.Kuttner, Note on strong summability, *J. London Math. Soc.*, **21**(1946), 118-122.

[24] J.Lindenstrauss and L.Tzafriri, On Orlicz sequence spaces, *Israel J. Math.*, **10** (1971), 379-390.

[25] I.J.Maddox, Sequence spaces defined by a modulus, *Math. Proc. Cambridge Philos. Soc*, **100(1)** (1986), 161-166.

[26] F.Moricz, Extentions of the spaces c and c_0 from single to double sequences, *Acta. Math. Hungerica*, **57(1-2)**, (1991), 129-136.

[27] F.Moricz and B.E.Rhoades, Almost convergence of double sequences and strong regularity of summability matrices, *Math. Proc. Camb. Phil. Soc.*, **104**, (1988), 283-294.

[28] M.Mursaleen,M.A.Khan and Qamaruddin, Difference sequence spaces defined by Orlicz functions, *Demonstratio Math.* , **Vol. XXXII** (1999), 145-150.

[29] M.Mursaleen and O.H.H. Edely, Statistical convergence of double sequences, *J. Math. Anal. Appl.*, **288(1)**, (2003), 223-231.

[30] M.Mursaleen, Almost strongly regular matrices and a core theorem for double sequences, *J. Math. Anal. Appl.*, **293(2)**, (2004), 523-531.

[31] M.Mursaleen and O.H.H. Edely,Almost convergence and a core theorem for double sequences, *J. Math. Anal. Appl.*, **293(2)**, (2004), 532-540.

[32] I.J.Maddox, On strong almost convergence, *Math. Proc. Cambridge Philos. Soc.*, **85(2)**, (1979), 345-350.

[33] H.Nakano, Concave modulars, *J. Math. Soc. Japan*, **5**(1953), 29-49.

[34] W.Orlicz, Über Raume $\left(L^M\right)$ *Bull. Int. Acad. Polon. Sci. A*, **(**1936), 93-107.

[35] S.D.Parashar and B.Choudhary, Sequence spaces defined by Orlicz functions, *Indian J. Pure Appl. Math.* , **25(4)**(1994), 419-428.

[36] A.Pringsheim, Zurtheorie derzweifach unendlichen zahlenfolgen, *Mathematische Annalen*, **53**, (1900), 289-321.

[37] W.H.Ruckle, FK spaces in which the sequence of coordinate vectors is bounded, *Canad. J. Math.*, **25**(1973), 973-978.

[38] , G.M.Robison, Divergent double sequences and series, *Amer. Math. Soc. Trans.*, **28**, (1926), 50-73.

[39] N.Subramanian, R.Nallswamy and N.Saivaraju, Characterization of entire sequences via double Orlicz space, *Internaional Journal of Mathematics and Mathemaical Sciences*, **Vol.2007**(2007), Article ID 59681, 10 pages.

[40] N.Subramanian and U.K.Misra, The semi normed space defined by a double gai sequence of modulus function, *Fasciculi Math.*, **46**, (2010).

[41] N.Subramanian and U.K.Misra, Characterization of gai sequences via double Orlicz space, *Southeast Asian Bulletin of Mathematics*, **(revised)**.

[42] N.Subramanian, B.C.Tripathy and C.Murugesan, The double sequence space of Γ^2, *Fasciculi Math.* , **40**, (2008), 91-103.

[43] N.Subramanian, B.C.Tripathy and C.Murugesan, The Cesáro of double entire sequences, *International Mathematical Forum*, **4 no.2**(2009), 49-59.

[44] N.Subramanian and U.K.Misra, The Generalized double of gai sequence spaces, *Fasciculi Math.*, **43**, (2010).

[45] N.Subramanian and U.K.Misra, Tensorial transformations of double gai sequence spaces, *International Journal of Computational and Mathematical Sciences*, **3:4**, (2009), 186-188.

[46] L.L.Silverman, On the definition of the sum of a divergent series, *un published thesis*, **University of Missouri studies, Mathematics series**.

[47] B.C.Tripathy, On statistically convergent double sequences, *Tamkang J. Math.*, **34(3)**, (2003), 231-237.

[48] B.C.Tripathy,M.Et and Y.Altin, Generalized difference sequence spaces defined by Orlicz function in a locally convex space, *J. Analysis and Applications*, **1(3)**(2003), 175-192.

[49] A.Turkmenoglu, Matrix transformation between some classes of double sequences, *Jour. Inst. of math. and Comp. Sci. (Math. Seri.)*, **12(1)**, (1999), 23-31.

[50] O.Toeplitz, *Ü*ber allgenmeine linear mittel bridungen, *Prace Matemalyczno Fizyczne (warsaw)*, **22**, (1911).

[51] A.Wilansky, Summability through Functional Analysis, *North-Holland Mathematics Studies, North-Holland Publishing, Amsterdam*, **Vol.85**(1984).

[52] M.Zeltser, Investigation of Double Sequence Spaces by Soft and Hard Analitical Methods, Dissertationes Mathematicae Universitatis Tartuensis 25, *Tartu University Press, Univ. of Tartu, Faculty of Mathematics and Computer Science, Tartu*, **2001**.

[53] C.R.Adams, Housdroff transformations for double sequences, *Bull. Amer. Math. Soc.*, **39**, (1933), 303-312.

[54] J.Qkutoyi and B.Thorpe, The spectrum of the Cesàro operator on $c_0\,(c_0)$, *Math. Proc. Cambridge Phil. Soc.*, **105**(1989)(MR 89i: 47051), 123-129.