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# ON THE BOUNDARIES OF CHANGING PARAMETERS IN THE MATHEMATICAL MODELING OF THE DYNAMIC SYSTEM OF THE FIGURE OF THE EARTH

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**Annotation:** In nature, the observed phenomena and processes in a given area of space for a given moment of time exist only in one copy. The observed facts are reflected in the form of statistical data. In mathematical modelling, the choice of parameters is arbitrary. Only in one set of sets of parameters does it correspond to the observed reality. Thus, mathematical modelling allows a more general analysis of the phenomena and processes observed in nature. In this paper, it is shown that the statistical patterns of the evolution of the elements of both the outer form and the internal structure of the Earth are combined in the form of motions of continents, with different velocities:

$$1\ sm/year \le \upsilon \le 9\ sm/year\ ; \quad (1972-1986\ \ y.y.) \qquad \text{(I)}$$

The time in which the observation results were obtained is indicated in brackets. Variations in the velocities of the continents (I) correspond to changes in the velocities of the mean radius of the vector of the expanding model of the figure of the Earth:

$$2\ sm/year \le dR/dt \le 3{,}6\ sm/year. \qquad \text{(II)}$$

and shrinking model

$$-0{,}304\, mm/year \le dR/dt \le -0{,}242\, mm/year. \qquad \text{(III)}$$

Other parameters of the Earth figure undergo similar changes. Under the conditions (I), (II) and (III), the internal structure and external shape of the Earth's figure retain their stable states. If the conditions (I), (II) and (III) are violated, they can go on to unstable states. The reason for this can be both external bombardment and internal and external resonant phenomena.

**Keywords:** Dynamic systems; mathematical modelling; the figure of the Earth; evolution; stability.

**1. Introduction.** In dynamic systems, the observations carried out cover a certain time interval $\Delta t = t_n - t_1$, where $t_1 -$ indicates the time of the first observation and $t_n -$ the time of n's observations.

Statistic data $a_1, a_2, ..., a_n$ correspond to points of time $t_1, t_2, ..., t_n$ . They obey the statistical laws of the problem under consideration. Based on these data and patterns, we perform mathematical modelling of the problem. Identification of this model and the real problem is carried out by means of a comparative analysis of statistical data and regularities of the real problem, covering the time interval $\Delta t$, with the results of calculation by formulas of the

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model problem, for the same instants of time. Thus, the basis of mathematical modelling is statistical data and the corresponding statistical regularities obtained on the basis of observations. The goal of mathematical modelling is to determine both the future and the distant past of the problem [1-12].

In [6], graphs for the variations of the coefficients $J_2$ are given. They are based on the observations of K. Christopher and Ch. Benjamin. Graphs of variations $J_n$ $(n \geq 2)$, that are based on the model of the Jacobi dynamic system, were constructed in [8; 9]. They show that models based on dynamic systems can reflect the evolution of real Earth processes correctly.

**2. Analysis of changes in the parameters of the figure of the Earth.** Statistical regularities refer only to estimates that are found from observations. We denote by $O(t_1), O(t_2),...., O(t_n) -$ numbers corresponding to statistical data, found from observations at times $t_1, t_2,..., t_n$; $C(t_1), C(t_2),...., C(t_n) -$ the numbers found from the calculations, from the formulas of mathematical modelling, for the same moments of time.The values $O(t_n)$ $u$ $C(t_n)$ satisfy inequation:

$$\sum_{n=1}^{\infty} R(t_n) \leq O(t_n) - C(t_n) \leq \sum_{n=1}^{k} F(t_n), \ (k = 1, 2, ...); \tag{1}$$

Where $\sum_{n=1}^{\infty} R(t_n)$ -the sum of infinitesimal perturbations (actions) that are elusive by direct measurements. They have a hidden effect on the values of observations and are regulators of sets of natural phenomena. $\sum_{n=1}^{k} F(t_n)$ - the sum of the forces acting, under the influence of which certain processes are happening that are captured by mathematical modelling.

The statistical data and patterns corresponding to them, over time, undergo various changes that correspond to disturbing actions.Therefore, unambiguous identification of both primary and final sources of information is impossible. They are associated with the prerequisites of mathematical modeling, and can be determined approximately. This is due to the elusive sum of infinitesimal perturbations and resonance phenomena between the elements of the dynamic system that continuously change with time, as well as the configuration of the elements of the system.

We note that the above assumptions and arguments apply only to stable systems. In all other cases, dynamic systems are under enormous pressure of numerous nonlinear fields surrounding them. In addition, they are influenced by various resonant phenomena. The evolution of these systems occurs under the influence of the sums of an infinite set of both small and large forces. They are able to change the stability of the system to instability, and vice versa. Thus, the evolution of dynamic systems as well as the connections between its elements depends on perturbations and resonance phenomena, both internal and external objects of the system.

Table 1 shows the speed of evolution of distances between the Earth's continents, during 1972-1986, according to [1, p.190]. They are defined in three different ways.

**Table1.** Speed of change of distances between Earth continents

| Distance (chord length) between | Increases with speed, (*sm/year*) |
|---|---|
| Europe and North America | $1{,}5 \pm 0{,}5$ |
| North America and Hawaii | $4{,}1 \pm 1$ |
| Hawaiiand South America | $5 \pm 3$ |
| South America and Australia | $6 \pm 3$ |
| AustraliaandHawaii | $7 \pm 1$ |

Errors: $\pm 0{,}5$; $\pm 1$; $\pm 3$; $\pm 3$; $\pm 1$, sm/year, are the sum of the errors of measuring instruments: time, distance, mass and the sum of infinitesimal actions (disturbances) of lesser or the same order.

Considering this table 1 can be rewritten as table 2.

**Table 2.** Speed of change of distances between Earth continents.

| Distance (chord length) between | Increases with speed $\upsilon$ |
|---|---|
| Europe and North America | $1 \le \upsilon \le 2$ *sm/year* |
| North America and Hawaii | $3{,}1 \le \upsilon \le 4{,}2$ *sm/year* |
| Hawaiiand South America | $2 \le \upsilon \le 8$ *sm/year* |
| South America and Australia | $3 \le \upsilon \le 9$ *sm/year* |
| Australia and Hawaii | $6 \le \upsilon \le 8$ *sm/year* |

From Table 2 it follows that the size, shape, and internal structure of the Earth, continuously changing with time and they go the way of the evolution of statistical patterns. Thus, the statistical patterns of the evolution of the elements of the outer form and the internal structure of the Earth are combined in the form of motions of continents with different velocities: $1 \leq \upsilon \leq 9\, sm/year$ . Changes in these boundaries, both in the past and in the future, can only be investigated by mathematical modelling. And this can be realized with the help of statistical estimates found from observations. All observed phenomena of nature reflect a set of specific statistical data and statistical regularities. The results of qualitative analysis of mathematical modelling depend on the time interval covered by the estimates of observations. In this case, one must not forget about the perturbations and evolution of various combinations of the elements of the system.

Estimations of the results of observations constitute only statistical regularities of the observed processes and phenomena. All the rest belongs to the model problem, which is formed on the basis of various simplifications and assumptions. To control the correctness of the model problem (mathematical modeling), it must satisfy known statistical data.

Collected by millennia of observation data, as well as the current state of the Earth allow for a different modeling of the evolution of both the external form and its internal structure. However, none of these models can cover completely the evolution of Earth processes and phenomena over geological time intervals.

According to [1], the velocity of the secular change in the mean radius of the Earth R of the expanding model of the Earth can be represented by the formula:

$$dR / dt = (2{,}8 \pm 0{,}8)\, sm / year, \tag{2}$$

i.e. varies between boundaries:

$$2\, sm/year \leq dR / dt \leq 3{,}6\, sm/year \tag{3}$$

The validity of these estimates is shown in three different ways. Laser measurements carried out using both 1) the surface of the Moon, and 2) artificial Earth satellites such as Lageos;
3) measurements of long-baseline interferometers.

In the works of Yu.B. Barkin [4; 5], in three different ways it is shown that the rate of secular changes in the mean radius of a compressible Earth model can be determined by the formula:

$$dR / dt = -(0{,}273 \mp 0{,}031) mm/year, \tag{4}$$

in other words, varies between boundaries: $-0{,}304 mm/year \leq dR / dt \leq -0{,}242 mm/year$ (5)

The mathematical model represents a certain number of formulas. They express the relationship between the measurable magnitude of the problem and the parameters of the proposed model. The accuracy of these formulas shows how much this model corresponds to reality. Relations between the parameters of expanding and contracting geodynamic models with measurable parameters of the Earth can be represented by the formulas:

$$I_n = 0{,}25\left(c^n / R^n\right)\cdot\left[(1+i\delta)(\delta+i)^n + (1-i\delta)(\delta-i)^n\right], \left(n\geq 2,\ i=\sqrt{-1}\right), \qquad (6)$$

where $I_n -$ parameters characterizing the structure of the Earth, $c$ and $\delta -$ parameters of the Jacobi dynamic system. When the first three terms of the expansion series of the Earth's potential are taken into account, the parameters $c$ и $\delta$ can be represented by the equalities:

$$c = \left\{-2I_2 - \left(\frac{I_3}{2I_2}\right)^2\right\}^{1/2} R; \qquad (7)\ \delta = \frac{I_3}{2I_2}\left\{-2I_2 - \left(\frac{I_3}{I_2}\right)^2\right\}^{-1/2}. \qquad (8)$$

The radius of the vectors and acceleration of gravity at various parallels of the Earth's surface, i.e. $\left[r_n(\varphi), g_n(\varphi)\right]$ are determined from observations.

The equation of the level surface corresponding to the secular variation of the average equatorial radius of the Earth (4) can be expressed by the formula:

$$\begin{aligned} r_n = &\left[R_0 - 2{,}73\cdot 10^{-5}(t-t_0)\right]\cdot\{1 - 1{,}5(I_3)_n \sin\varphi + 2{,}5(I_3)_n\cdot\sin^3\varphi - \\ &-0{,}5\left[q - 3(I_2)_n + 7{,}5(I_3)_n - 4{,}5(I_2)_n^2 + q^2\right]\sin^2\varphi + \\ &+0{,}75\left[q^2 - 3{,}5(\gamma_2) - 6(I_2)^2\right]\sin^4\varphi + ... \end{aligned} \qquad (9)$$

where $q = \omega^2 R_0^2 /(fm) = 0{,}00346142$.

Secular changes in the acceleration of gravity $g_n$, depending on geographical latitude $\varphi$ on a surface $r_n$,

$$\begin{aligned} g_n = &\frac{fm}{R_0^2}\left[1 - \frac{2{,}23\cdot 10^{-3}}{R_0^2}\cdot(t-t_0)\right]^{-2}\cdot\{q1{,}5(I_2)_n + 1{,}875(I_3)_n - 3(I_3)_n\sin\varphi + \\ &+\left[2q(1+q^2) + 1{,}5(I_2)_n(1-5q) + 9(I_2)_n^2 - 11{,}25(I_3)_n\right]^2\sin^2\varphi + \\ &+5(I_3)_n\sin^3\varphi\left[1 - 2{,}75q^2 + \left(7{,}5q(I_2)_n\ 2{,}25(I_2)_n^2 + 16{,}625(I_3)_n\right]\sin^4\varphi\right\}... \end{aligned} \qquad (10)$$

Similarly $r_n = r_n(\varphi,t)$ and formulas $g_n = g_n(\varphi,t)$ can easily be obtained for the expanding geodynamic, mathematical model of the Earth[7].

Using the statistical data given in [10], from (9) we obtain:

$$\left(r_n' - r_n''\right)(O-C)\leq 1m\ ;\left(r_0 - r_n'\right)(O-C)\leq 13m\,;\left(r_0 - r_n''\right)(O-C)\leq 15m \qquad (11)$$

where $r_n' = r_0(90°) -$ distance from the center of mass of the Earth to its north pole, $\left(\varphi = 90°\right)$; $r_n'' = r_n(-90°) -$ distance from the center of mass of the Earth to its south pole,

$(\varphi = -90°)$; $r_0 = r_n(0°) -$ average radius of the equator of the Earth, $(\varphi = 0°)$. In this case, we used as the observed values:

$$O(r_n' - r_n'') = 32\,m; \quad O(r_0' - r_n') = 21373\,m; \; O(r_0 - r_n'') = 21466\,m. \tag{12}$$

The compression of the middle, northern and southern hemispheres of the level surface (9) is equal to:

$$\alpha = 1 - \frac{r_n' - r_n''}{2r_0} = 1/298{,}12; \; \alpha' = \frac{r_0 - r_n'}{r} = 1/298{,}42; \alpha'' = \frac{r_0 - r_n''}{r_0} = 1/297{,}96 \tag{13}$$

In addition, from the expression (10) we find:

$$-5{,}186 \le g_n' - g_0 \le 5{,}230 \; gal; \quad -5{,}180 \le g_n'' - g_0 \le 5{,}230 \; gal, \tag{14}$$

Where $g_n' = g_n(90°)$; $g_n'' = g_n(-90°)$ и $g_0 = g_n(0°) -$ acceleration of gravity at the northern, southern poles and equator of the Earth, respectively.

The differences (14) correspond to the experimental values, which are determined from the direct measurements and are equal $\pm 5\,гал$ [11] .

The observed state of the Earth is the result of the total effects and evolution of numerous actions (currents). Therefore, the accuracy of observations, the time of observations, and also the duration of time intervals of observations play an important role in all the dynamic processes of the formation and evolution of the Earth. All these factors are included in the statistical estimates of observations. At the same time, in dynamic models, many actions and consequences can not be taken into account i.e. their sum is less than the values of statistical estimates that represent the errors of observations.

**3. Conclusion.**Thus, the comparison of results by the formulas of geodynamic mathematical models show that both flattening in the poles and expansion in the equator of the Earth can be attributed to the number of real phenomena of nature.

Differences $O(t_n) - C(t_n)$, $(n = 1, 2, \ldots k)$ play a key role, to understanding: the distant past, the analysis of the future and the phenomena observed today. However, they may not reflect the actual reality of the process, since the statistical data used in the calculations, found on the basis of observations covering only a limited time interval.

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