International Journal of Science, Environment
and Technology, Vol. 1, No 2, 49-55, 2012

SOME NEW PARANORMED SEQUENCE SPACES DEFINED BY ORLICZ FUNCTION

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Abstract. In this paper we present new classes of sequence spaces using the concept of n-norm and to investigate these spaces for some linear topological structures as well as examine these spaces with respect to derived (n-1) norms. We use an Orlicz function, a bounded sequence of positive real numbers and Λ_m operator to construct these spaces so that they became more generalized. This investigations will enhance the acceptability of the notion of n-norm by giving a way to contruct different sequence spaces with elements in n-normed space.

Key words and phrases: n-norm, paranorm, completeness, Orlicz function.

AMS Subject Classification: 40A05, 46A45, 46B70

1. INTRODUCTION

Recall in [6] that an Orlicz function M is continuous, convex, nondecreasing function define for $x > 0$ such that $M(0) = 0$ and $M(x) > 0$. If convexity of Orlicz function is replaced by $M(x+y) \leq M(x) + M(y)$ then this function is called the modulus function and characterized by Ruckle [7] .An Orlicz function M is said to satisfy $\Delta_2 - condition$ for all values u, if there exists $K > 0$ such that $M(2u) \leq KM(u), u \geq 0$.

Lemma. Let M be an Orlicz function which satisfies $\Delta_2 - condition$ and let $0 < \delta < 1$. Then for each $t \geq \delta$, we have $M(t) < K\delta^{-1}M(2)$ for some constant $K > 0$.

A sequence space X is said to be solid or normal if $(\alpha_k x_k) \in X$, and for all double sequences $\alpha = (\alpha_k)$ of scalars with $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$.

The concept of 2-normed spaces was initially developed by Gahler [5] in the mid of 1960's, while that of n-normed spaces can be found in Misiak [4]. Since then, many others have studied this concept and obtained various results, see for instance Gunawan $[2-3]$, Gunawan and Mashadi [1], Esi $[9-10]$, Esi and Ozdemir [11], Fistikci and et al.[12] and many others.

Let $n \in \mathbb{N}$ and X be a real vector space of dimension d, where $n \leq d$. A real-valued function $\|., ..., .\|$ on X satisfying the following four condition:

(i) $\|x_1, x_2, ..., x_n\| = 0$ if and only if $x_1, x_2, ..., x_n$ are linearly dependent,

(ii) $\|x_1, x_2, ..., x_n\|$ is invariant under permutation,

(iii) $\|\alpha x_1, x_2, ..., x_n\| = |\alpha| \|x_1, x_2, ..., x_n\|, \alpha \in \mathbb{R}$,

(iv) $\|x_1 + x_1^{\imath}, x_2, ..., x_n\| \leq \|x_1, x_2, ..., x_n\| + \|x_1^{\imath}, x_2, ..., x_n\|$

called an n-norm on X, and the pair $(X, \|., ..., .\|)$ is called an n-normed space [2] .

Let $(X, \|., ..., .\|)$ be an n-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, ..., a_n\}$ be a linearly independent set in X. Then the following function $\|., ..., .\|_\infty$ on X^{n-1}

defined by

$$\|x_1, x_2, ..., x_{n-1}\|_\infty = \max\{\|x_1, x_2, ..., x_{n-1}, a_i\| : i = 1, 2, ..., n\}$$

defines an (n-1)-norm on X with respect to $\{a_1, a_2, ..., a_n\}$.

Let $n \in \mathbb{N}$ and $(X, \langle ., . \rangle)$ be a real inner product space of dimension $d \geq n$. Then the following function $\|., ..., .\|_S$ on $X \times X \times ... \times X$ (n factors) defined by

$$\|x_1, x_2, ..., x_n\|_S = [\det(\langle x_i, x_j \rangle)]^{\frac{1}{2}}$$

is an n-norm on X, which is known as standard n-norm on X. If we take $X = \mathbb{R}^n$, then this n-norm is exactly the same as Euclidean n-norm such as

$$\|x_1, x_2, ..., x_n\|_E = abs\left(\begin{vmatrix} x_{11}...x_{1n} \\ ... \\ x_{n1}...x_{nn} \end{vmatrix}\right)$$

where $x_i = (x_{i1}, ..., x_{in}) \in \mathbb{R}^n$ for each i=1,2,...,n.

We procure the following results those will help in establishing some results of this article.

Lemma 1.[1] A standard n-normed space is complete if and only if it is complete with respect to the usual norm $\|.\| = \langle ., . \rangle^{\frac{1}{2}}$.

Lemma 2.[1] On a standard n-normed space X, the derived (n-1)-norms $\|., ..., .\|_\infty$, defined with respect to orthonormal set $\{e_1, e_2, ..., e_n\}$, is equivalent to the standard (n-1)-norms $\|., ..., .\|_S$. Precisely, we have for all $x_1, x_2, ..., x_{n-1}$

$$\|x_1, x_2, ..., x_{n-1}\|_\infty \leq \|x_1, x_2, ..., x_{n-1}\|_S \leq \sqrt{n}\,\|x_1, x_2, ..., x_{n-1}\|_\infty$$

where $\|x_1, x_2, ..., x_{n-1}\|_\infty = \max\{\|x_1, x_2, ..., x_{n-1}, e_i\| : i = 1, 2, ..., n\}$.

In paper [8], Mursaleen and Noman introduced the notion of $\lambda-$*convergent* and $\lambda-$*bounded* sequences as follows: Let $\lambda = (\lambda_k)_{k=0}^{\infty}$ be a strictly increasing sequence of positive real numbers tending to infinity, that is

$$0 < \lambda_o < \lambda_1 < ... \text{ and } \lambda_k \to \infty \text{ as } k \to \infty$$

and said that a sequence $x = (x_k) \in w$ is $\lambda-$*convergent* to the number L, called a the $\lambda-$limit of x, if $\Lambda_m(x) \to L$ as $m \to \infty$, where

$$\Lambda_m(x) = \frac{1}{\lambda_m}\sum_{k=1}^{m}(\lambda_k - \lambda_{k-1})\, x_k.$$

The sequence $x = (x_k) \in w$ is $\lambda-$*bounded* if $\sup_m |\Lambda_m(x)| < \infty$. It is well known [8] that if $\lim_m x_m = a$ in the ordinary sense of convergence, then

$$\lim_m \left(\frac{1}{\lambda_m}\sum_{k=1}^{m}(\lambda_k - \lambda_{k-1})\,|x_k - a|\right) = 0.$$

This implies that

$$\lim_m |\Lambda_m(x) - a| = \lim_m \left|\frac{1}{\lambda_m}\sum_{k=1}^{m}(\lambda_k - \lambda_{k-1})(x_k - a)\right| = 0$$

which yields that $\lim_m \Lambda_m(x) = a$ and hence $x = (x_k) \in w$ is $\lambda-$*convergent* to a.

2. MAIN RESULTS

Let $(X, \|., ..., .\|)$ be real n-normed space and $w(n - X)$ denotes the space of X-valued sequences. Let M be an Orlicz function and $p = (p_k)$ be any bounded sequence of strictly positive real numbers. Now, we define the following sequence spaces:

$$[M, \Lambda, p, \|., ..., .\|]_o = \left\{ \begin{array}{c} x = (x_k) \in w(n - X) : \ \lim_m \left[M \left(\left\| \frac{\Lambda_m(x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} = 0 \\ for \text{ some } \rho > 0 \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \end{array} \right\},$$

$$[M, \Lambda, p, \|., ..., .\|] = \left\{ \begin{array}{c} x = (x_k) \in w(n - X) : \lim_m \left[M \left(\left\| \frac{\Lambda_m(x) - L}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} = 0 \\ for \text{ some } \rho > 0, L \in X \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \end{array} \right\},$$

and

$$[M, \Lambda, p, \|., ..., .\|]_\infty = \left\{ \begin{array}{c} x = (x_k) \in w(n - X) : \ \sup_m \left[M \left(\left\| \frac{\Lambda_m(x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} < \infty \\ for \text{ some } \rho > 0 \text{ and for every } z_1, z_2, ..., z_{n-1} \in X \end{array} \right\}$$

The following well-known inequality will be used in this study: If $0 \leq \inf_k p_k = H_o \leq p_k \leq \sup_k = H < \infty$, $D = \max\left(1, 2^{H-1}\right)$, then

$$|x_k + y_k|^{p_k} \leq D \left\{ |x_k|^{p_k} + |y_k|^{p_k} \right\}$$

for all $k \in \mathbb{N}$ and $x_k, y_k \in \mathbb{C}$. Also $|x_k|^{p_k} \leq \max\left(1, |x_k|^H\right)$ for all $x_k \in \mathbb{C}$.

In this section we investigate some linear topological structures of the sequence spaces $[M, \Lambda, p, \|., ..., .\|]_o$, $[M, \Lambda, p, \|., ..., .\|]$ and $[M, \Lambda, p, \|., ..., .\|]_\infty$.

It is clear from the definition that $[M, \Lambda, p, \|., ..., .\|]_o \subset [M, \Lambda, p, \|., ..., .\|]$. Further $[M, \Lambda, p, \|., ..., .\|] \subset [M, \Lambda, p, \|., ..., .\|]_\infty$ follows from the following inequality

$$\left[M \left(\left\| \frac{\Lambda_m(x)}{2\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m}$$
$$\leq \left[\frac{1}{2} M \left(\left\| \frac{\Lambda_m(x) - L}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} + \left[\frac{1}{2} M \left(\left\| \frac{L}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m}.$$

Similarly, we have $[M, \Lambda, p, \|., ..., .\|]_o \subset [M, \Lambda, p, \|., ..., .\|] \subset [M, \Lambda, p, \|., ..., .\|]_\infty$.

Theorem 2.1. If $\{\Lambda_m(x), z_1, z_2, ..., z_{n-1}\}$ is a linearly dependent set in $(X, \|., ..., .\|)$ for all but finite m, where $x = (x_k) \in w(n - X)$ and $\inf_m p_m > 0$, then

(i) $\lim_m \left[M \left(\left\| \frac{\Lambda_m(x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} = 0$, for every $\rho > 0$,

(ii) $\sup_m \left[M \left(\left\| \frac{\Lambda_m(x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} < \infty$, for every $\rho > 0$.

Proof.(i). Suppose that $\{\Lambda_m(x), z_1, z_2, ..., z_{n-1}\}$ is linearly dependent set in $(X, \|., ..., .\|)$ for all but finite m. Then we have

$$\|\Lambda_m(x), z_1, z_2, ..., z_{n-1}\| \to 0 \ as \ m \to \infty.$$

Since M is continuous and $\inf_m p_m > 0$ for all m, we have

$$\lim_m \left[M \left(\left\| \frac{\Lambda_m(x)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right]^{p_m} = 0, \ for \ every \ \rho > 0.$$

(ii) The proof is similar to part (i).

Theorem 2.2. The classes of sequences $[M,\Lambda,p,\|.,...,.\|]_o$, $[M,\Lambda,p,\|.,...,.\|]$ and $[M,\Lambda,p,\|.,...,.\|]_\infty$ are linear spaces.

Proof. We will prove only for $[M,\Lambda,p,\|.,...,.\|]_\infty$ and the others can be proved similarly. Let $x,y\in[M,\Lambda,p,\|.,...,.\|]_\infty$. Then there exist $\rho_1>0$ and $\rho_2>0$ such that

$$\left[M\left(\left\|\frac{\Lambda_m(x)}{\rho_1},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_m}<\infty$$

and

$$\left[M\left(\left\|\frac{\Lambda_m(y)}{\rho_2},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_m}<\infty$$

for all $m\geq 1$. Let α,β be any scalars and let $\rho_3=\max(2|\alpha|\rho_1,2|\beta|\rho_2)$. Then we have

$$\left[M\left(\left\|\frac{\Lambda_m(\alpha x+\beta y)}{\rho_3},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_m}$$
$$\leq\left[M\left(\left\|\frac{\Lambda_m(\alpha x)}{\rho_3},z_1,z_2,...,z_{n-1}\right\|\right)+M\left(\left\|\frac{\Lambda_m(\beta y)}{\rho_3},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_m}$$
$$\leq D\left\{\left[M\left(\left\|\frac{\Lambda_m(x)}{\rho_1},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_m}+\left[M\left(\left\|\frac{\Lambda_m(y)}{\rho_2},z_1,z_2,...,z_{n-1}\right\|\right)\right]^{p_m}\right\}$$

for all $m\geq 1$. Hence $[M,\Lambda,p,\|.,...,.\|]_\infty$ is a linear space.

Theorem 2.3. The spaces $[M,\Lambda,p,\|.,...,.\|]_o$, $[M,\Lambda,p,\|.,...,.\|]$ and $[M,\Lambda,p,\|.,...,.\|]_\infty$ are complete paranormed spaces, paranormed by h defined by

$$h(x)=\inf\left\{\rho^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m(x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\},$$

where $H=\max(1,\sup_m p_m)$.

Proof. Clearly $h(x)=h(-x)$ and $h(\theta)=0$. Let $x=(x_k)$ and $y=(y_k)$ be any two sequences belong to any one of the spaces $[M,\Lambda,p,\|.,...,.\|]_o$, $[M,\Lambda,p,\|.,...,.\|]$ and $[M,\Lambda,p,\|.,...,.\|]_\infty$. Then we have $\rho_1>0$ and $\rho_2>0$ such that

$$\sup_m\left[M\left(\left\|\frac{\Lambda_m(x)}{\rho_1},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1$$

and

$$\sup_m\left[M\left(\left\|\frac{\Lambda_m(y)}{\rho_2},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1.$$

Let $\rho=\rho_1+\rho_2$. Then by the convexity of M, we have

$$\sup_m\left[M\left(\left\|\frac{\Lambda_m(x+y)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]$$
$$\leq\left(\frac{\rho_1}{\rho_1+\rho_2}\right)\sup_m\left[M\left(\left\|\frac{\Lambda_m(x)}{\rho_1},z_1,z_2,...,z_{n-1}\right\|\right)\right]$$
$$+\left(\frac{\rho_2}{\rho_1+\rho_2}\right)\sup_m\left[M\left(\left\|\frac{\Lambda_m(y)}{\rho_2},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1.$$

Hence we have

$$h(x+y)=\inf\left\{\rho^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m(x+y)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}$$
$$\leq\inf\left\{\rho_1^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m(x)}{\rho_1},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}$$

$$+\inf\left\{\rho_2^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m(y)}{\rho_2},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}.$$

This implies $h(x+y)\leq h(x)+h(y)$. The continuity of the scalar multiplication follows from the following equality:

$$h(\alpha x)=\inf\left\{\rho^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m(\alpha x)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}$$

$$=\inf\left\{(t\,|\alpha|)^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m(x)}{t},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\},$$

where $t=\frac{\rho}{|\alpha|}$. Now let $\left(x^i\right)$ be any Cauchy sequence in any one of the spaces $[M,\Lambda,p,\|.,...,.\|]_o$, $[M,\Lambda,p,\|.,...,.\|]$ and $[M,\Lambda,p,\|.,...,.\|]_\infty$, where $x^i=\left(x_o^{(i)},x_1^{(i)},x_2^{(i)},...\right)$. Let $x_o>0$ be fixed and $t>0$ be such that for a given ε $(0<\varepsilon<1)$, $\frac{\varepsilon}{x_ot}>0$ and $x_ot\geq 1$. Then there exists a positive integer $n_o(\varepsilon)$ such that

$$h\left(x^i-x^j\right)<\frac{\varepsilon}{x_ot}\quad\text{for all } i,j\geq n_o.$$

Using the definition of paranorm, we get

$$\inf\left\{\rho^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m\left(x^i-x^j\right)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}$$

$$<\frac{\varepsilon}{x_ot}\quad\text{for all } i,j\geq n_o.$$

Then we have

$$\inf\left\{\rho^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m\left(x^i-x^j\right)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}<\varepsilon\quad\text{for all } i,j\geq n_o.$$

Hence, we have

$$\sup_m\left[M\left(\left\|\frac{\Lambda_m\left(x^i-x^j\right)}{h\left(x^i-x^j\right)},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\quad\text{for all } i,j\geq n_o.$$

It follows that

$$M\left(\left\|\frac{\Lambda_m\left(x^i-x^j\right)}{h\left(x^i-x^j\right)},z_1,z_2,...,z_{n-1}\right\|\right)\leq 1\quad\text{for each } m\geq 1 \text{ and for all } i,j\geq n_o.$$

For $t>0$ with $M\left(\frac{x_ot}{2}\right)\geq 1$, we have

$$M\left(\left\|\frac{\Lambda_m\left(x^i-x^j\right)}{h\left(x^i-x^j\right)},z_1,z_2,...,z_{n-1}\right\|\right)\leq M\left(\frac{x_ot}{2}\right).$$

Then we have

$$\left\|\Lambda_m\left(x^i-x^j\right),z_1,z_2,...,z_{n-1}\right\|\leq\frac{x_ot}{2}.\frac{\varepsilon}{x_ot}=\frac{\varepsilon}{2},\ \text{for all } i,j\geq n_o,$$

which leads to the fact that $\left(\Lambda_mx^o,\Lambda_mx^1,\Lambda_mx^2,...\right)$ is a Cauchy sequence in X for all $m\in\mathbb{N}$. Since X is complete then it is convergent. Let $\lim_i\Lambda_m\left(x^i\right)=\Lambda_m(x)$. Now we have for all $i,j\geq n_o$

$$\inf\left\{\rho^{\frac{p_m}{H}}:\sup_m\left[M\left(\left\|\frac{\Lambda_m\left(x^i-x^j\right)}{\rho},z_1,z_2,...,z_{n-1}\right\|\right)\right]\leq 1\right\}<\varepsilon.$$

This implies that

$$\lim_j \inf \left\{ \rho^{\frac{p_m}{H}} : \sup_m \left[M \left(\left\| \frac{\Lambda_m \left(x^i - x^j \right)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right] \leq 1 \right\} < \varepsilon.$$

Since M and n-norms are continuous functions, we have

$$\inf \left\{ \rho^{\frac{p_m}{H}} : \sup_m \left[M \left(\left\| \frac{\Lambda_m \left(x^i - x \right)}{\rho}, z_1, z_2, ..., z_{n-1} \right\| \right) \right] \leq 1 \right\} < \varepsilon, \text{ for all } i \geq n_o.$$

It follows that $\left(x^i - x\right)$ belongs to any one of the spaces $[M, \Lambda, p, \|., ..., .\|]_o$, $[M, \Lambda, p, \|., ..., .\|]$ and $[M, \Lambda, p, \|., ..., .\|]_\infty$. Since these spaces are linear, so we have $x = x^i - \left(x^i - x\right)$ belongs to any one of the spaces. This completes the proof.

We state the following Theorem in view of Lemma 2.

Theorem 2.4. Let X be a standard n-norm space and $\{e_1, e_2, ..., e_n\}$ be an orthonormal set in X. Then

$$[M, \Lambda, p, \|., ..., .\|_\infty]_o = \left[M, \Lambda, p, \|., ..., .\|_{(n-1)}\right]_o,$$

$$[M, \Lambda, p, \|., ..., .\|_\infty] = \left[M, \Lambda, p, \|., ..., .\|_{(n-1)}\right]$$

and

$$[M, \Lambda, p, \|., ..., .\|_\infty]_\infty = \left[M, \Lambda, p, \|., ..., .\|_{(n-1)}\right]_\infty$$

where $\|., ..., .\|_\infty$ is the derived (n-1)-norm defined with respect to $\{e_1, e_2, ..., e_n\}$ and $\|., ..., .\|_{(n-1)}$ is the standard (n-1)-norm on X.

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Received 11 April, 2012