

## GENERALISED HEAD-DISCHARGE-INCLINATION RELATIONSHIP FOR FLOW THROUGH INCLINED TRIANGULAR WEIR WITH NEW APPROACH

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**Abstract:** A weir is an obstruction put across a flow such that the head above the crest (fluid level on the upstream side) is measured and the quantity of flow can be estimated. Weirs are also flow control devices. The sharp crested weirs with its more accurate and consistent performance are usually used in laboratories rather in the field channels for flow measurements due to its major drawback in development of afflux. In particular, Triangular weir has advantages about rectangular weirs. It can increase the discharge relatively better than rectangular weir with increased head. To reduce the afflux, researchers have attempted placing weirs inclined to the bed of the channel. This procedure of practice of the thin plate weirs of standard geometric shape has increased the discharging coefficient. This paper is concerned with the experimentation, analysis and development of discharge-head-inclination model to measure flow over inclined triangular weir with a new approach. The new method is simple and the error analysis shows that the estimated discharge is well within 2% and less than the previous methods of flow analysis. The research work is of considerable academic interest in showing that the head index is independent of weir inclination and depends on the shape of the weir opening. It is shown that the discharge coefficient increases with increase in weir angle with respect to normal (vertical) position, thereby reducing the afflux which may prompt field channel users to implement these weirs with simple head-discharge-inclination equation. The weir position of  $75^\circ$  with respect to vertical plane along the flow direction has been experimented and analysed.

**Keywords:** Thin plate Weirs; Inclined weirs, Triangular weirs, Discharge characteristics, Flow measurement, Afflux, Inclined-weir discharging index.

### INTRODUCTION

A weir is an obstruction put across a flow such that the head above the crest (fluid level on the upstream side) is measured and the quantity of flow can be estimated. Weirs are also flow control devices. Sharp crested weirs do have a very short length of the order of 1 to 2mm crest length along the flow direction and have distinct characteristics relative to broad crested weirs. The sharp crested weirs with its more accurate and consistent performance are usually used in laboratories rather in the field channels for flow measurements. A number of investigations on weir profiles have been done by many researchers for flow measurements

mainly to reduce the afflux and to increase the discharging capacity. Normally the weirs are fixed normal to the flow axis. However to gain some benefits in the flow measurements, these weirs can also be arranged oblique to the flow axis known as oblique weir and side weirs. The literature in this paper is restricted to analysis of flow through the inclined weirs.

Tullis et al (1) studied Labyrinth weir having trapezoidal plan forms and presented results in the form of curves between discharge coefficient and  $E/w$  with angle ' $\theta$ ' as the third parameter.

Talib Mansoor (2) has developed equation for elementary discharge coefficient for rectangular skew weirs.

Shesha Prakash and Shivapur (3,4,5,6,7,8,9,10) investigated about the discharge coefficient relationship with the angle of inclination with respect to the normal (standard) position of plane of weir for a sharp crested triangular weir. They have analysed the flow through weirs as a vector and resolved the flow through inclined weir as horizontal and vertical components and computed the net discharge as combined discharge. They have proposed a coefficient  $\beta$  to be a function of weir inclination angle  $\alpha$  to develop a general equation for flow through inclined sharp crested triangular weirs. However there was no rational explanation for the use of coefficient  $\beta$  in the generalised discharge equation which was empirical. Most of the researchers have reported their study on establishing discharge coefficient in terms of weir head to weir height/length and skew angle. The majority of these investigators have used other type of sharp crested weirs to calibrate the weir under investigation, limiting the accuracy of the results obtained.

Shesha Prakash et.al (11,12,13,14,15,16,17,18,19,20) have worked on flow through inclined weirs with different openings like Rectangular, triangular, trapezoidal and Inverted V-Notch (IVN). They have developed a mathematical model for the flow through weirs and software to compute the head-discharge equation for the given type of weir and inclination.

In both the cases the results depended on the modeling and the both coefficients, viz., weir discharge coefficient and head index were developed on the basis of experimental values.

In the present study, a different approach is followed to obtain the weir discharge coefficient for flow through inclined triangular weir.

As it is evident from Fig.1, the flow length along the plane of the weir increases with increase in inclination with the normal plane which increases the discharging capacity of the weir.

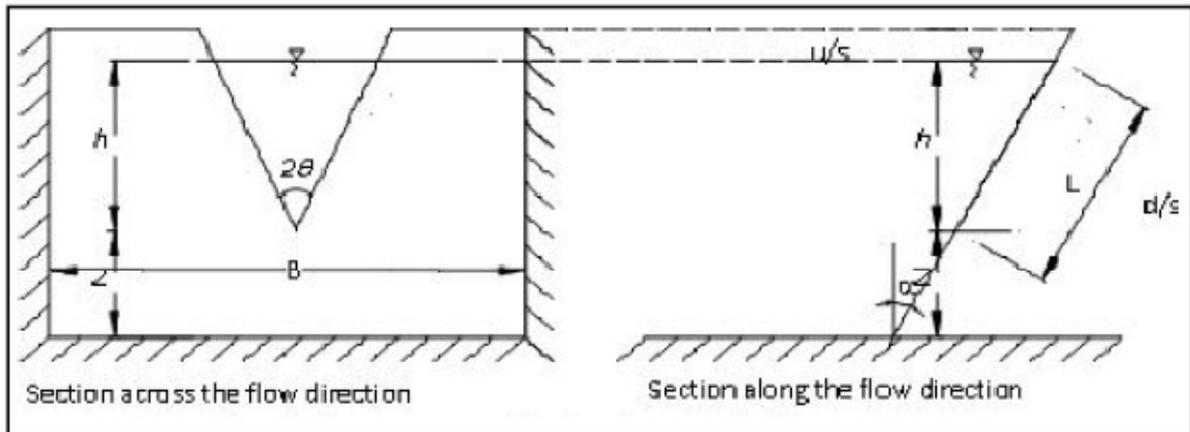


Fig1. Arrangement of Inclined Triangular Weir

### Formulation of the Problem:

The discharge for flow through triangular weir given by Darcy-Weisbach equation is

$$q = \frac{8}{15} \sqrt{2g} \tan \theta h^{\frac{5}{2}} \quad (01)$$

Initially the obtained and computed values of Head and discharge were non-dimensionalised as below so that obtained equation will be more generic in nature. Where q is the discharge through the weir in  $\text{m}^3/\text{s}$ , and h is the head over the crest in m.

Non-dimensionalising the above equation, we get

$$Q = \left[ \frac{8}{15} \right] \tan \theta H^{\frac{5}{2}}$$

where  $Q = \frac{q}{\sqrt{2gW^{\frac{5}{2}}}}$  and  $H = \frac{h}{w}$  (02)

### Experiments

Experiments were carried on inclined triangular weir fixed normal to the flow direction ( $0^\circ$ ),  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ ,  $75^\circ$  inclinations with respect to the normal plane (Vertical) along the flow axis. The experimental channel is rectangular in section and having dimensions 0.3m wide, 0.2m deep and of 4.6m length. The channel is constructed of Perspex sheet and has smooth walls and nearly horizontal bed to reduce the boundary frictional force. It is connected to a Head tank of capacity 750 liters. The inclined triangular weir is made of 8mm stainless steel with a crest thickness of 1 mm and a  $45^\circ$  chamfer given on downstream side to get a springing nappe. The experimental set up is shown in Fig 2.



**Fig.2** Experimental Setup

Water is supplied to the channel by an inlet valve provided on supply pipe. Overhead tank is provided with overflow arrangement to maintain constant head. Smooth, undisturbed, steady-uniform flow was obtained by making the water to flow through graded aggregates and the surface waves were dampened by tying gunny bags at the surface near the tank. The head over the weir is measured using an electronic point gauge placed in piezometer located at a distance of about 1.40m on upstream of inclined rectangular notch. A collecting tank of size 1 m length, 0.6 m breadth, and of 0.6 m depth is provided with a piezometer. Water after running through the experimental setup is collected in a sump from which it is re-circulated by pump by lifting it back to the overhead tank

In the present study, the conventional method of volumetric discharge measurement is used, which increases the accuracy of the work. The measurements are done through electronic point gauge which automatically detects the water level and records the gauge reading. The volumetric measurement is done through self-regulated timer for a fixed rise of water level automated through sensors. To eliminate the human error in piezometric readings in the collecting tank, time was auto recorded by an electronic timer, for the predefined interval of rise of water in the tank and averaged, by considering the cumulative volume and the accumulated time.

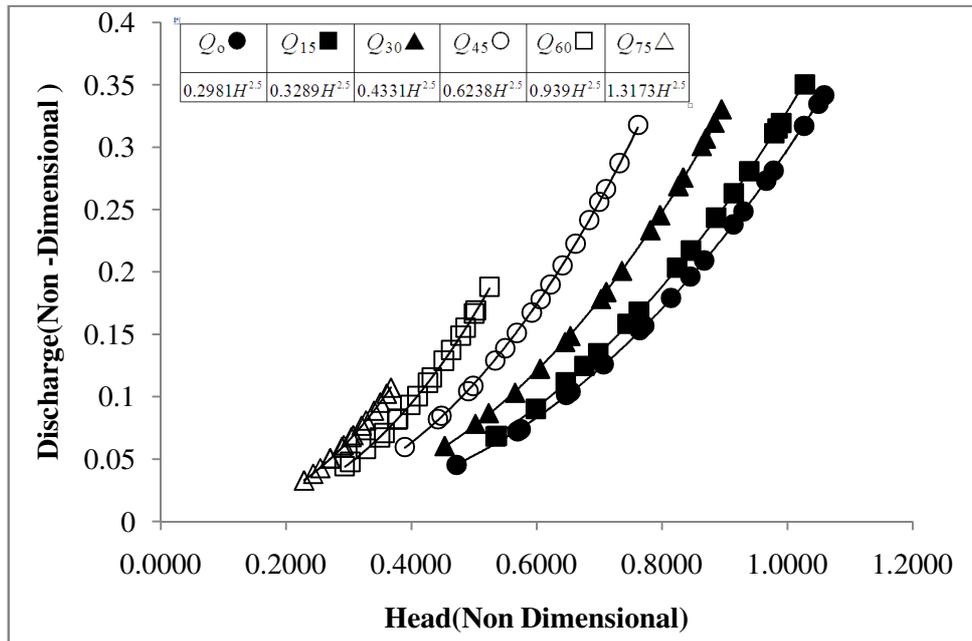
The present investigation was carried out on the range of variables shown in Table1.

**Table-1:** Range of variables studied.

Position of the weir	Normal	Angle of inclination with vertical plane in degrees				
	0	15	30	45	60	75
Head over the crest (m)	28.34	32.06	27.18	23.38	17.62	13.71
	63.52	61.68	53.69	45.70	31.46	22.03
Actual Discharge	0.1771	0.2662	0.2356	0.2326	0.1726	0.1274
	1.3338	1.3681	1.2902	1.2410	0.7339	0.4180
No. of runs	40	36	32	28	21	24

### Analysis of results

A plot of Non-dimensional discharge versus non-dimensional head for various positions of plane of triangular weir have been shown in Fig.3. It shows that the discharge increases with increase in inclination angle  $\alpha$ .



**Fig 3:** Non-dimensional Head-discharge plot for various values of inclination  $\alpha$ .

Hence, sharp-crested triangular weir can be installed at a suitable inclination in the channel without any alteration to the conventional simple geometry of the weir so that the discharging capacity of the weir can be much higher, corresponding to the same head, as compared to conventional normal weir, which is evident from Fig. 3. This will help in reducing free board

requirement on upstream of weir position. Further, it can also be used in the existing channels with least effect of afflux.

### Mathematical Modeling:

Cowgill and Banks have shown that the head-discharge equation is a function of weir profile equation and proved that  $Q \propto H^{(1+\frac{1}{2})} \Rightarrow \phi H^{\frac{3}{2}}$ , the profile equation for head-discharge equation of relationship  $Q \propto H^n$  will be given by  $y \propto x^{(n-\frac{1}{2})}$ . From Cowgill and Banks, for a rectangular weir with profile equation  $y \propto x$ , the head-discharge equation will be

$$Q \propto H^{(1+\frac{1}{2})} \Rightarrow \phi H^{\frac{3}{2}},$$

where  $\phi$  is the discharge coefficient for the weir.

With this it can be seen that the Head index is a function of weir profile and is assumed to be nearly constant at 1.5. Hence the discharge coefficient will be a function of weir inclination *ai.e.*  $\phi=f(\alpha)$ .

The discharge-head-inclination equation can be expressed as

$$Q = f(\alpha)H^{1.5} \tag{03}$$

This method reduces the complicated two dimensional variation of weir discharging index and head index to simple weir discharging index.

A programmable algorithm is used to obtain a second order polynomial to get the inclined-weir-discharging index as follows:

The modeling part is subdivided into two stages.

Initially the Actual and theoretical discharge values for corresponding weir inclinations are tabulated as  $Q_{ai}$  and  $Q_{ti}$ . The corresponding weir discharging index for given inclination is found by the following equation:

$$\phi_i = \frac{\Sigma Q_{ai}^2}{\Sigma Q_{ai} Q_{ti}}$$

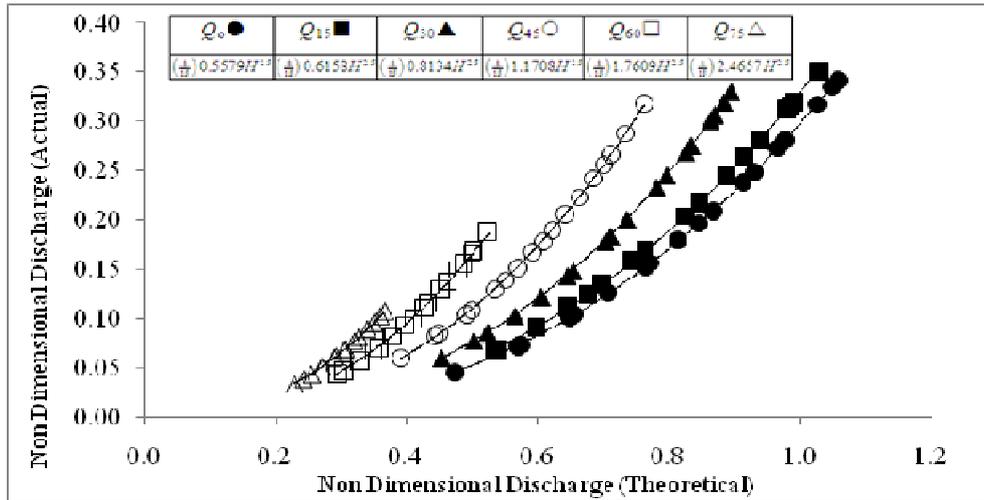


Fig. 4: Variation of Non-dimensional Actual and Theoretical Discharge for various weir inclinations

The modeling consists of directly getting the inclined weir discharging index by performing regression analysis directly on non-dimensional, actual and theoretical discharges for corresponding inclinations. Fig. 4 shows the variation of Non-dimensional actual discharge with theoretical discharge for corresponding inclinations. As the theoretical discharge equation is derived for normal position of the weir, the coefficient will be obtained as an index of the corresponding inclined head-discharge equation with respect to normal position, thereby giving a better weir discharging index ( $\phi$ ). The retention of  $\left(\frac{8}{15}\right)$  in the derivation of non-dimensional head-discharge equation indicates that the concerned weir is triangular weir and easily the obtained results can be compared with other sharp crested weirs.

$$\phi_i = \frac{(Q_a)_i}{(Q_t)_n}$$

The head-discharge equation for flow through any sharp crested weir is given by the relation

$$Q = \phi H^n$$

**Table-2:** Calibrated Head-discharge equations for various angles  $\alpha$ .

Legend/Method	$Q_0 \bullet$	$Q_{15} \blacksquare$	$Q_{30} \blacktriangle$	$Q_{45} \circ$	$Q_{60} \square$	$Q_{75} \triangle$
SheshaPrakash et.al.	$0.2981H^{2.5}$	$0.3289H^{2.5}$	$0.4331H^{2.5}$	$0.6238H^{2.5}$	$0.939H^{2.5}$	$1.3173H^{2.5}$
Presented Method	$\left(\frac{8}{15}\right)0.5579H^{2.5}$	$\left(\frac{8}{15}\right)0.6158H^{2.5}$	$\left(\frac{8}{15}\right)0.8134H^{2.5}$	$\left(\frac{8}{15}\right)1.1708H^{2.5}$	$\left(\frac{8}{15}\right)1.7609H^{2.5}$	$\left(\frac{8}{15}\right)2.4657H^{2.5}$

In the second phase the obtained 6 weir-discharging coefficients are listed against weir inclinations as radians relatively as  $y_i$  and  $x_i$  and arranged as matrix to develop the model for

the present problem, 2<sup>nd</sup> order polynomial curve can be fit to the data and simplifying the equations, we get the final general head-discharge-angle expression for any given rectangle, of and inclination as under:

$$\begin{bmatrix} 6 & \Sigma x_i & \Sigma x_i^2 \\ \Sigma x_i & \Sigma x_i^2 & \Sigma x_i^3 \\ \Sigma x_i^2 & \Sigma x_i^3 & \Sigma x_i^4 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \\ \Sigma x_i^2 y_i \end{bmatrix}$$

Using the data obtained,

$\Sigma x_i = 3.927$	$\Sigma y_i = 7.385$
$\Sigma x_i^2 = 3.770$	$\Sigma x_i y_i = 6.578$
$\Sigma x_i^3 = 4.037$	$\Sigma x_i^2 y_i = 7.144$
$\Sigma x_i^4 = 4.599$	$n = 6$

Substituting the values, we get

$$\begin{bmatrix} 6 & 3.927 & 3.770 \\ 3.927 & 3.770 & 4.037 \\ 3.770 & 4.037 & 4.599 \end{bmatrix} \begin{bmatrix} \alpha^2 \\ \alpha \\ C \end{bmatrix} = \begin{bmatrix} 7.385 \\ 6.578 \\ 7.144 \end{bmatrix}$$

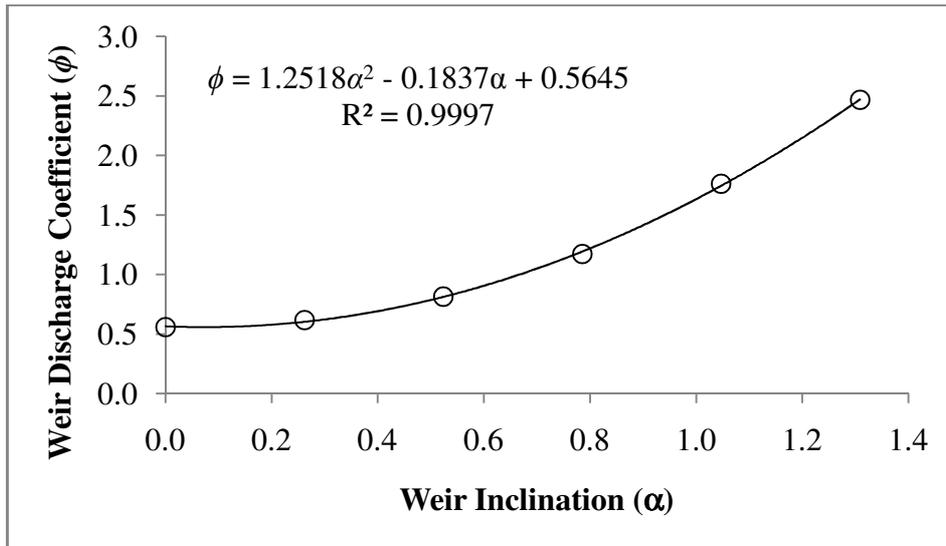
Solving by Gaussian elimination method, we get the quadratic equation as

$$\phi = 1.2518\alpha^2 - 0.1837\alpha + 0.5645$$

$$Q = (1.2518\alpha^2 - 0.1837\alpha + 0.5645)H^{(\frac{5}{2})}$$

$$\text{where } Q = \frac{q}{\sqrt{2gW^{\frac{5}{2}}}}; H = \frac{h}{w} \text{ and } \alpha \text{ is in radians}$$

The same is obtained as an Excel curve fit as shown in Fig. 5.



**Fig. 5:** Variation of weir discharge coefficient ( $\phi$ ) with weir inclination ( $\alpha$ )

It can be seen that the equation developed by the model agrees with the one obtained by Excel and further, the regression coefficient in both the cases is exactly unity. This improves the credibility of the analysis and practical usage of the notch. Even though obtained discharge-head-inclination equation is complicated, it reduces to simple equation once the  $\alpha$  values are substituted and simplified.

#### **Error analysis:**

Error analysis is carried out by computing the percentage deviation of the Computed discharge from the actual discharge for various inclinations for method developed by Shesha Prakash et. al and the presented method as shown in Table-3.

**Table-3:** Maximum absolute percentage deviation of Computed to Actual discharge.

Legend/Method	Normal	Angle of inclination with vertical plane in degrees				
		0	15	30	45	60
Shesha Prakash et.al.	1.00	1.02	1.05	0.22	0.99	1.09
Present Research	1.17	1.15	1.17	0.00	1.12	1.16

#### **Discharging capacity:**

The discharging capacity of the inclined triangular weir relative to normal position is shown in Fig. 6. *Inclined weir discharging index* is the ratio of the discharging capacity of an inclined weir to that of normal weir. *Inclined weir discharging index* for inclined triangular weir for various inclinations with respect to normal position of triangular weir are as plotted in Fig. 5.

$$C_{di} = 2.2437\alpha^2 - 0.3292\alpha + 0.0118$$

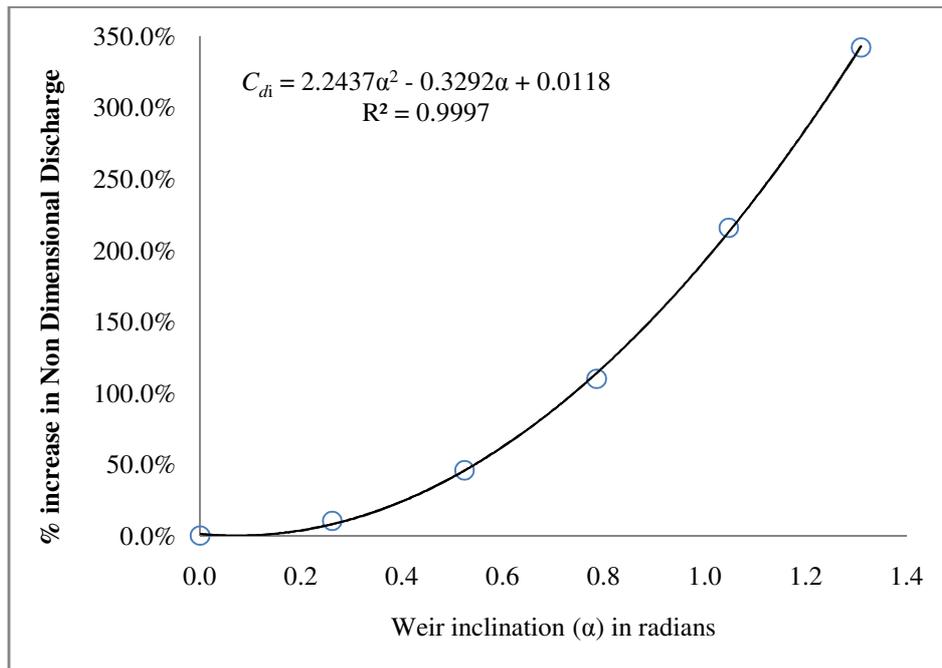


Fig. 6 Variation of Inclined weir discharging index with weir inclination

#### Analysis of afflux:

The Nondimensional Head is computed for the Maximum Nondimensional Discharge for various inclinations of the triangular notch as shown in Table 4. It can be seen from Table 4 that the maximum reduction is with 75° inclination.

**Table 4:** Reduction in Afflux and increase in discharging capacity for various inclinations of Triangular notch relative to its normal position.

$\alpha$		$Q_{\max} = 1.37E-03$ LPS		$H = \left[ \frac{Q}{\phi} \right]^{\frac{1}{n}}$	%ge Reduction in Afflux	Discharge per unit head	%ge increase in $Q$
Deg	Radians	$\alpha$	$n$				
0	0.00	0.558	2.5	0.09	0.0%	0.558	0.0%
15	0.26	0.616	2.5	0.087	3.3%	0.616	10.4%
30	0.52	0.813	2.5	0.078	13.3%	0.813	45.8%
45	0.79	1.171	2.5	0.067	25.6%	1.171	109.9%
60	1.05	1.761	2.5	0.057	36.7%	1.761	215.6%
75	1.31	2.466	2.5	0.05	44.4%	2.466	342.0%

### Conclusions

Following conclusions were drawn based on the experimental investigation and the subsequent analysis by the authors.

- The discharging capacity of the weir increases with the increase in inclination of the plane of weir. In particular it is found to increase exponentially from 15° to 75°.
- It is observed from Table3 that the percentage deviation of error in computation of discharge is well within 2% for all angles of inclination of the weir.
- Larger flow area is possible in the inclined weirs relative to the conventional normally positioned weirs. From Table4, it is seen that afflux is found to be about 44.4% with 75° inclination of triangular weir relative to its normal position. The property of increase in inclined weir discharging index with increase in inclination of weir plane can be explored to discharge more water quickly without increasing afflux on upstream side in predesigned canal structure during flood season, without changing the position of the preinstalled weir (which is practically very difficult). It is found to be 342.0% increase with 75° weir inclination relative to its normal position.
- In the presented analysis, the retention of fraction  $\left(\frac{8}{15}\right)$  indicating the feature of head-discharge relation for triangular weir will render the comparison of the obtained parametric values with other type of weirs more reasonable.
- Due to the simple geometry and ease of construction, inclined triangular weirs find its applications as a simple measuring devices in irrigation, chemical and sanitary engineering for flow measurement and flow control.

- The Mathematical modeling results in a single head discharge inclination equation which can be used for any triangular weir of any desired inclination.

**Limitation:**

The experiment can be done with larger discharge in larger channels and the Head Discharge Inclination equation can be improved by using the model.

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