International Journal of Science, Environment and Technology, Vol. 4, No 4, 2015, 1073 – 1079

ISSN 2278-3687 (O)
2277-663X (P)

STUDY OF THERMAL EFFECT ON VIBRATION OF VISCO-ELASTIC RECTANGULAR PLATE OF LINEARLY VARYING THICKNESS UNDER THERMAL CONDITION

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Abstract: A mathematical model is presented here to study the free vibrations of homogeneous rectangular plate of variable thickness which is clamped at all four edges. The main objective of the present study is to help the engineers and practitioners in designing various structures in the field of science and technology. The analysis is presented here to study two directional thermal effects with bi-linearly varying thickness. Rayleigh-Ritz method is used to calculate time period for first two modes of vibration for different values of thermal gradient, taper constants and aspect ratio. All the results are presented in form of tables for an alloy of Aluminium, ***'Duralumin'***.
Keywords: Vibration, thermal gradient, taper constant, clamped, aspect ratio.

1. INTRODUCTION

Structures of plates are commonly used in the field of science and technology i.e. ships, aircrafts, bridges, etc. The dynamic study of the behavior and characteristics of vibration of plate is essential to assess and use the full potential of plates. In the engineering, we cannot neglect the analysis of effect of vibration because almost all machines and engineering structures experiences vibrations.

Leissa [1] discussed different models on free vibration of rectangular plates. Jain and Soni [2] studied the free vibrations of rectangular plates with parabolically varying thickness. Singh and Saxena [4] studied the transverse vibration of rectangular plate with bi-directional thickness variation. Leissa [5] discussed the historical bases of the Rayleigh-Ritz Methods. Li [6] analyzed the vibration of rectangular plate with general elastic boundary supports. Lal [9] et al. studied the transverse vibrations of non-homogeneous rectangular plates with varying thickness. Khanna et al. [10] discussed the thermal effect on the vibration of a non-homogeneous rectangular plate with varying poisson ratio.

*Received July 3, 2015 * Published Aug 2, 2015 * www.ijset.net*

Here, authors analyzed vibration of rectangular plate with non-uniform thickness. Non-uniformity in thickness is considered bi-linear. Also, the temperature variation is considered bi-parabolic. Authors calculated the time period for first two modes of vibration at various values of thermal gradient, taper constant and aspect ratio for the clamped boundary condition at all four edges. Results are given in tabular form.

2. MATERIALS AND METHODS

2.1 Analysis of Motion

Equation of motion for isotropic rectangular plate in Cartesian coordinate is [12]:

$$\widetilde{D}\Bigg[D_1\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+2\frac{\partial^3 w}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+2\frac{\partial^3 w}{\partial y\,\partial x^2}\right)$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\upsilon\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\upsilon\frac{\partial^2 w}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\Bigg]+\rho h\frac{\partial^2 w}{\partial t^2}=0 \quad (2.1.1)$$

where $D_1=\frac{Eh^3}{12(1-\upsilon^2)}$ is the flexural rigidity of the plate's material, $\widetilde{D}$ is visco-elastic operator , $W = W(x, y)$ is the deflection function, υ is poisson ratio, ρ is mass per unit volume and h is thickness of the plate.

Taking deflection w as a product of two functions [11] as:

$$w(x, y, t) = W(x, y).T(t) \quad (2.1.2)$$

where T(t) is a time function.

Substituting the equation (2.1.2) into equation (2.1.1), one obtains

$$\Bigg[D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+2\frac{\partial^3 W}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+2\frac{\partial^3 W}{\partial y\,\partial x^2}\right)$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\upsilon\frac{\partial^2 W}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\Bigg]/\,\rho hW=-\left(\frac{\partial^2 T/\partial t^2}{\widetilde{D}T}\right)$$

(2.1.3)

Taking both sides of equation (2.1.3) equals to a constant p^2, we have

$$\Bigg[D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+2\frac{\partial^3 W}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+2\frac{\partial^3 W}{\partial y\,\partial x^2}\right)$$

$$+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\upsilon\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\upsilon\frac{\partial^2 W}{\partial x^2}\right)+2(1-\upsilon)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\Bigg]-\rho p^2 hW=0 \quad (2.1.4)$$

and $\frac{\partial^2 T}{\partial t^2} + p^2 \widetilde{D} T = 0$, (2.1.5)

These are the differential equations of motion (2.1.4) and time function (2.1.5) for rectangular plate of variable thickness in Cartesian coordinate respectively.

It is assumed that thickness of the rectangular plate varies linearly in both directions [7] i.e.

$$h = h_0 \left(1 + \beta_1 \frac{x}{a}\right)\left(1 + \beta_2 \frac{y}{b}\right) \tag{2.1.6}$$

where a & b are length and breadth of rectangular plate respectively and β_1 & β_2 are taper parameters in x-direction and y-direction respectively.

Authors also assumed bi-parabolic temperature variations as:

$$\tau = \tau_0 \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right) \tag{2.1.7}$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature excess above the reference temperature at $x = y = 0$.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed as follows [3]:

$$E = E_0(1 - \gamma\tau) \tag{2.1.8}$$

where E_0 is value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is slope of variation of E and τ. On using equation (2.1.7) in equation (2.1.8), one obtains

$$E = E_0 \left[1 - \alpha\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right] \tag{2.1.9}$$

where, $\alpha = \gamma\tau_0$ $(0 \leq \alpha < 1)$ is thermal gradient.

On substituting the values of h and E from equations (2.1.6) and (2.1.9), the expression of flexural rigidity (D_1) becomes:

$$D_1 = \frac{E_0\left[1 - \alpha\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)\right] h_0^3 \left(1 + \beta_1 \frac{x}{a}\right)^3 \left(1 + \beta_2 \frac{y}{b}\right)^3}{12(1 - \upsilon^2)} \tag{2.1.10}$$

2. 2 Solution of Frequency Equation

Rayleigh Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy (S_E) must be equal to the maximum kinetic energy (K_E). So it is necessary for the problem under consideration that [8]

$$\delta(S_E - K_E) = 0 \tag{2.2.1}$$

Now assuming the non-dimensional variables as

$$X = \frac{x}{a},\ \ Y = \frac{y}{a},\ \ \overline{W} = \frac{w}{a},\ \ \overline{h} = \frac{h}{a}, \tag{2.2.2}$$

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$W = \frac{\partial W}{\partial x} = 0 \ \text{ at } x = 0,\ a \ \text{ and } \ W = \frac{\partial W}{\partial y} = 0 \ \text{ at } y = 0,\ a \tag{2.2.3}$$

Corresponding two-term deflection function can be taken as [10]

$$W = \left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2 \times \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \tag{2.2.4}$$

The expressions for kinetic energy (K_E) and strain energy (S_E) are

$$K_E = \frac{1}{2}\rho p^2 \overline{h}_0 a^5 \int_0^1 \int_0^{b/a} \left[(1+\beta_1 X)\left(1+\beta_2 \frac{a}{b} Y\right)\overline{W}^2\right] dY\, dX \tag{2.2.5}$$

$$\& \ S_E = \frac{E_0 \overline{h}_0^3 a^3}{24(1-\upsilon^2)} \int_0^1 \int_0^{b/a} \left\{1-\alpha(1-X^2)\left(1-\frac{a^2}{b^2}Y^2\right)\right\}(1+\beta_1 X)^3 \left(1+\beta_2 \frac{a}{b}Y\right)^3 \times \left[\left(\frac{\partial^2 \overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2 \overline{W}}{\partial Y^2}\right)^2 + 2\upsilon \frac{\partial^2 \overline{W}}{\partial X^2}\frac{\partial^2 \overline{W}}{\partial Y^2} + 2(1-\upsilon)\left(\frac{\partial^2 \overline{W}}{\partial X \partial Y}\right)^2\right] dY dX \tag{2.2.6}$$

On using equations (2.2.5) & (2.2.6) in equation (2.2.1), one gets

$$\delta(S_E^* - \lambda^2 K_E^*) = 0 \tag{2.2.7}$$

Where $$K_E^* = \int_0^1 \int_0^{b/a} \left[(1+\beta_1 X)\left(1+\beta_2 \frac{a}{b}Y\right)\overline{W}^2\right] dY\, dX \tag{2.2.8}$$

$$\& \ S_E^* = \int_0^1 \int_0^{b/a} \left\{1-\alpha(1-X^2)\left(1-\frac{a^2}{b^2}Y^2\right)\right\}(1+\beta_1 X)^3 \left(1+\beta_2 \frac{a}{b}Y\right)^3 \times \left[\left(\frac{\partial^2 \overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2 \overline{W}}{\partial Y^2}\right)^2 + 2\upsilon \frac{\partial^2 \overline{W}}{\partial X^2}\frac{\partial^2 \overline{W}}{\partial Y^2} + 2(1-\upsilon)\left(\frac{\partial^2 \overline{W}}{\partial X \partial Y}\right)^2\right] dY dX \tag{2.2.9}$$

Here, $\lambda^2 = \dfrac{12\rho p^2 a^2 (1-\upsilon^2)}{E_0 \overline{h}_0^2}$ is a frequency parameter.

Equation (2.2.4) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of W. These two constants are to be determined as follows:

$$\frac{\partial}{\partial A_n}\left(S_E^* - \lambda^2 K_E^*\right) = 0, \quad n = 1, 2 \tag{2.2.10}$$

On simplifying equation (2.2.10), one gets

$$b_{n1}A_1 + b_{n2}A_2 = 0,\ n = 1, 2 \qquad (2.2.11)$$

where b_{n1} and b_{n2} include parametric constant.

With the help of the solution of equation (2.2.11), one can obtains a quadratic equation in p^2 from which the two values of p^2 can be found easily.

Time period of the vibration of the plate is given by: $K = \frac{2\pi}{p}$ (2.2.12)

3. RESULTS AND DISCUSSION

In the calculations, the following parameters are used for duralumin:

$E = 7.08\times10^{10} N/m^2$, $G=2.632 \times 10^{10}\ N/m^2$, $\eta = 14.612 \times 10^5\ Ns/m^2$ and $\rho = 2.8 \times 10^3\ kg/m^3$

$\upsilon=0.345$, $h_0=0.01\ m$.

Table 3.1 and 3.2 contain the numerical values of time period for the first two mode of vibration for different values of taper constants (β_1 & β_2). It is seen that as taper constants increase, time period decreases.

Table 3.1

Time Period Vs Taper Constants at α = 0.0, a / b = 1.5						
β_2 →	**0**		**0.4**		**0.8**	
β_1 ↓	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0	169.01	667.93	138.39	547.54	114.83	455.59
0.4	139.94	552.71	114.59	453.15	95.09	377.15
0.8	118.48	467.50	97.03	383.39	80.53	319.23

Table 3.2

Time Period Vs Taper Constants at α = 0.4, a / b = 1.5						
β_2 →	**0**		**0.4**		**0.8**	
β_1 ↓	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0	186.91	739.25	150.26	596.24	123.19	491.55
0.4	153.76	607.59	123.72	490.72	101.54	404.97
0.8	129.59	511.59	104.37	413.69	85.72	341.75

Table 3.3

Time Period Vs Thermal Gradient at a / b = 1.5						
α ↓	**$\beta_1 = \beta_2 = 0.0$**		**$\beta_1 = \beta_2 = 0.4$**		**$\beta_1 = \beta_2 = 0.8$**	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0	169.01	667.93	114.59	453.15	80.53	319.23
0.2	177.29	700.83	118.89	470.70	83.01	329.80
0.4	186.91	739.25	123.72	490.72	85.72	341.75
0.6	198.29	784.98	129.19	513.89	88.71	355.41
0.8	212.08	840.74	135.44	541.24	92.02	371.32

Table 3.4

Time Period Vs Aspect Ratio						
a / b ↓	**$\alpha = \beta_1 = \beta_2 = 0.0$**		**$\alpha = \beta_1 = \beta_2 = 0.4$**		**$\alpha = \beta_1 = \beta_2 = 0.8$**	
	Mode 1	**Mode 2**	**Mode 1**	**Mode 2**	**Mode 1**	**Mode 2**
0.5	412.60	1650.14	301.66	1212.49	223.88	917.59
1	288.48	1128.99	211.42	829.33	157.56	627.45
1.5	169.01	667.93	123.72	490.72	92.02	371.32
2	103.15	412.53	75.41	303.12	55.97	229.39
2.5	68.08	274.43	49.73	201.66	36.86	152.62

Table 3.3 and 3.4 has the results of time period for the different values of thermal gradient and aspect ratio respectively. It is clear that time period increases when thermal gradient (α) increases and it decrease as aspect ratio (*a*/*b*) increases.

5. CONCLUSIONS

The main emphasis of the present study is to provide some numerical data about the first few modes of vibration for engineers and researchers to fulfill their requirements. The authors suggest that the readers and engineers can obtain the desired values of time period by appropriate tapering of the plate and can provide more reliable structures and designs.

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