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A STUDY OF THE TIME DEPENDENCE OF DECELERATION PARAMETER AND GRAVITATIONAL CONSTANT, ON THE BASIS OF BRANS-DICKE THEORY

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Abstract: A simple ansatz has been chosen for the Brans-Dicke (BD) scalar field ϕ where it has been shown to have the same dependence upon scale factor as that of the density of matter (ρ) of the universe. The present model is based on the generalized BD theory where the BD parameter ω is regarded as a function of the scalar field ϕ. Solving the field equations for a spatially flat Robertson-Walker space-time, the functional forms of $a(t)$, $q(t)$, $H(t)$, $G(t)$ have been determined and their inter-dependence has been analyzed in detail. The parameter $\omega(\varphi)$ has been found to have a negative value. The possibility of an inter-conversion between dark energy and matter has been taken into account by introducing a slowly varying function. A signature flip of deceleration parameter and an increase of gravitational constant with time have been found in the present study. The time dependence and inter-dependence of the relevant parameters have been explored both analytically as well as through numerical plots.
Keywords: Brans-Dicke theory; Accelerated expansion of the universe; Gravitational constant; Dark energy; Brans-Dicke scalar field; Signature flip of deceleration parameter; Cosmology.

Introduction

Recent observations regarding the expansion of the universe confirm that the universe has undergone a smooth transition from a decelerated to an accelerated phase of expansion [1,2]. Since normal matter has a positive definite density and pressure and gravitates in the usual manner, there must be some other kind of matter responsible for the observed acceleration, which makes the effective pressure sufficiently negative and gives rise to a repulsive effect. Such kind of matter is popularly known to be the 'Dark Energy'. Much attention has been devoted in past years to analyze the nature of it. A long list of models has evolved with an attempt to explain the acceleration to be ascertained.

Cosmological constant Λ is the simplest choice for the dark energy [3]. CDM model has a serious drawback in connection to the value of cosmological constant Λ. The currently observed value of Cosmological constant Λ for an accelerating Universe does not match with

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that of the value in Planck scale or Electroweak scale [4]. The problem can be rendered less acute if one tries to construct dark energy models with a time dependent cosmological parameter. Many such models have been proposed but they have their own problems [5, 6].

A suitable alternative to the dynamical Λ models are the scalar field models in which the equation of state of dark energy changes with time. Among the many proposed scalar field models, quintessence models are the ones endowed with a potential so that the contribution to the pressure sector, $p = {}^{1}/_{2}\,\dot{\varphi}^{2} - V(\varphi)$, can evolve to attain an adequately large negative value, thus generating the observed cosmic acceleration [7, 8]. One main drawback of these quintessence models is that most of the quintessence potentials are chosen arbitrarily and do not have a proper theoretical justification explaining their genesis. Naturally a large number of other alternative scalar field models, for example the tachyon [9, 10], k-essence [11, 12], holographic [13, 14] dark energy models have appeared in the literature with their own virtues and shortcomings.

In most of the scalar field models the cold dark matter and dark energy are normally allowed to evolve independently. However, there are attempts to include an interaction amongst them so that one grows at the expense of the other [15]. Non minimal coupling of the scalar field with the dark matter sector through an interference term in the action has helped in explaining the cosmic acceleration. Such fields are called 'Chameleon fields' and they have proved to be useful in playing the role of dark energy candidates [16, 17].

Non minimal coupling between the scalar field and geometry, especially in the frame work of Brans-Dicke theory, also pose themselves as possible candidates for explaining the observed acceleration. Modification of the Brans-Dicke (BD) theory by adding a potential $V(\varphi)$, which is a function of the BD scalar field φ itself, can serve as models explaining the acceleration of the Universe [18].

BD theory of cosmology has been analyzed with the aid of different models. To name a few, Sheykhi et al. [19] worked with the power-law entropy-corrected version of BD theory defined by a scalar field and a coupling function. In another literature Sheykhi et al. [20] considered the HDE model in BD theory to think about the BD scalar field as a possible candidate for producing cosmic acceleration without invoking auxiliary fields or exotic matter considering the logarithmic correction to the entropy. Jamil et. al. [21] studied the cosmic evolution in Brans-Dicke chameleon cosmology. Pasqua and Khomenko [22] studied the interacting logarithmic entropy-corrected HDE model in BD cosmology with IR cut-off given by the average radius of the Ricci scalar curvature.

Some models have also been suggested where a quintessence scalar field introduced in the BD theory can give rise to a late time acceleration for a wide range of potentials [23]. An interaction between dark matter and the BD scalar field showed that the matter dominated era can have a transition from a decelerated to an accelerated expansion without any additional potential [24]. On the other hand BD scalar field alone can also drive the acceleration without any quintessence matter or any interaction between BD field and dark matter [25].

However, the problem with many of these models is that the matter dominated Universe has an ever accelerating expansion contrary to the observations. Besides this, in order to explain the recent acceleration many of the models require a very low value of the BD parameter ω of the order of unity whereas the local astronomical experiments demand a very high value of ω [26].

Among the two different models of Brans-Dicke theory, one with a constant value of BD parameter ω, and the other with a variable Brans-Dicke parameter which is regarded as a function of the scalar field parameter φ, we have chosen the second one, known as generalized Brans-Dicke theory [27]. Incorporating an empirical function to account for an inter-conversion between matter and dark energy, we have determined the time dependence of all relevant cosmological parameters and shown their behaviour graphically.

Theoretical Model

For a spatially flat Robertson-Walker space-time, the field equations in the generalized Brans-Dicke theory are [27]

$$3\left(\frac{\dot{a}}{a}\right)^2=\frac{\rho}{\varphi}+\frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2-3\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}, \quad (1)$$

$$2\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2=-\frac{\omega(\varphi)}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2-2\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}-\frac{\ddot{\varphi}}{\varphi}. \quad (2)$$

Combining (1) and (2) one gets,

$$2\frac{\ddot{a}}{a}+4\left(\frac{\dot{a}}{a}\right)^2=\frac{\rho}{\varphi}-5\frac{\dot{a}}{a}\frac{\dot{\varphi}}{\varphi}-\frac{\ddot{\varphi}}{\varphi}. \quad (3)$$

Considering the conservation of matter of the universe we propose the following relation.

$$\rho=f(t)(\rho_0 a_0^3)a^{-3}=f(t)\rho_0 a^{-3}, \quad (a_0=1) \quad (4)$$

Here a_0 and ρ_0 are the scale factor and the matter density of the universe respectively at the present time. The reason for introducing the factor $f(t)$ is that the matter content of the universe may not remain proportional to $\rho_0 a_0^3$ [31]. There may be an inter-conversion between dark energy and matter (both baryonic and dark matter). It is assumed here that this conversion, if there is any, is extremely slow. In the present model therefore, the factor $f(t)$

is taken as a very slowly varying function of time, in comparison with the scale factor, and $f(t) = 1$ at $t = t_0$ where t_0 denotes the present instant of time when the scale factor $a = a_0 = 1$.

To make the differential equation (3) tractable, let us propose the following ansatz.

$$\varphi = \varphi_0 a^{-3} \tag{5}$$

This choice of φ makes the first term on the right hand side of equation (3) independent of the scale factor (a). In equation (5) we have taken $\varphi = \varphi_0$ for $a = a_0 = 1$.

Combining (3) and (5) and treating f as a constant we have,

$$\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a}\right)^2 = -f\frac{\rho_0}{\varphi_0} \tag{6}$$

In terms of Hubble parameter $H = \frac{\dot{a}}{a}$, equation (6) takes the following form.

$$\left(\dot{H} + H^2\right) - H^2 = \frac{dH}{dt} = -f\frac{\rho_0}{\varphi_0} \tag{7}$$

Integrating equation (7) and taking $H = H_0$ at $a = a_0 = 1$,

$$H = \frac{\dot{a}}{a} = f\frac{\rho_0}{\varphi_0}(t_0 - t) + H_0 \tag{8}$$

Integrating (8) and requiring that $a = a_0 = 1$ at $t = t_0$,

$$a = Exp\left[-\frac{1}{2}f\frac{\rho_0}{\varphi_0}(t^2 + {t_0}^2) + \left(f\frac{\rho_0}{\varphi_0}t_0 + H_0\right)t - H_0 t_0\right] \tag{9}$$

Using (9), the deceleration parameter $q = -\frac{\ddot{a}a}{\dot{a}^2}$ becomes

$$q = -1 + \frac{f\rho_0/\varphi_0}{\left(\frac{f\rho_0}{\varphi_0}(t_0 - t) + H_0\right)^2} . \tag{10}$$

Above equation clearly shows that a signature flip in q takes place at $t = \tau$ where,

$$\tau = t_0 - \left(\sqrt{\frac{\varphi_0}{f\rho_0}} - H_0\frac{\varphi_0}{f\rho_0}\right) \tag{11}$$

According to other studies $< t_0$. For this condition to be satisfied we must have,

$$f > \frac{H_0^2\varphi_0}{\rho_0} = 28.7266. \tag{11a}$$

The values of different cosmological parameters used in this study are,

$H_0 = 72\left(\frac{Km}{Sec}\right) per\ Mega\ Parsec\ = 2.33 \times 10^{-18} sec^{-1}$, $t_0 = 14\ billion\ years =$ $4.415 \times 10^{17} sec$, $\varphi_0 = \frac{1}{G} = 1.498 \times 10^{10} m^{-3} Kg s^2$, $\rho_0 = 2.831 \times 10^{-27} Kg/m^3$ (present density of dark matter + ordinary matter).

Let us now formulate the factor $f(t)$ from different criteria to be satisfied by it.

Based on (11a), we may take,

$f(t) = \alpha \frac{H_0^2 \varphi_0}{\rho_0}$ *with,* $\alpha > 1$ at $at\, t = \tau = \beta t_0$ with $\beta < 1$ (11b)

According to an initial assumption we have, $f(t) = 1\; at\; t = t_0$ (11c)

Let us now propose a linear relation between f and t which will satisfy the conditions expressed by (11b) and (11c). This relation is given by,

$$f = 1 + \left(\alpha \frac{H_0^2 \varphi_0}{\rho_0} - 1\right) \frac{t_0 - t}{t_0 - \beta t_0} \quad (11d)$$

Now letting $q = q_0$ at $t = t_0$ in (10), one obtains $q_0 = -1 + \frac{\rho_0}{H_0^2 \varphi_0} = -0.9652$.

Its negative sign shows that the universe is presently passing through a state of accelerated expansion.

The scale factor has been plotted as a function of time in figure 1.

Figure 2 has the plot of deceleration parameter as a function of time. It clearly shows that the universe is presently in a state of accelerated expansion. It made a transition from an initial accelerated state to a decelerated state before attaining the present state of acceleration.

According to Brans-Dicke theory, the gravitational constant is the reciprocal of the scalar field parameter φ. Therefore, using equations (5) and (9) we have,

$$G = \frac{1}{\varphi} = \frac{a^3}{\varphi_0} = \frac{1}{\varphi_0} Exp\left[-\frac{3}{2}\frac{f\rho_0 t^2}{\varphi_0} + 3\left(H_0 + \frac{f\rho_0 t_0}{\varphi_0}\right)t - 3H_0 t_0 - \frac{3}{2}\frac{f\rho_0 {t_0}^2}{\varphi_0}\right] \quad (12)$$

and the fractional change of G per unit time is given by,

$$\frac{\dot{G}}{G} = 3\left[\frac{f\rho_0}{\varphi_0}(t_0 - t) + H_0\right] \quad (13)$$

Equation (13) shows that, at the present time $(t = t_0)$, $\frac{\dot{G}}{G} = 3H_0 = 2.2 \times 10^{-10}\; Yr^{-1}$.

According to a study by Weinberg, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} \leq 4 \times 10^{-10}\; Yr^{-1}$ [33]. Our result is consistent with this observation.

In the figures (3) and (4), we have plotted the time variation of G and $\frac{\dot{G}}{G}$ respectively. The gravitational constant is found to increase with time with a varying rate. Both curves show that the universe is presently passing through a stage where the rate of G variation is the smallest. This increasing nature of G has been found in some other studies [28, 29, 30, 32].

At $t = t_0$, $\frac{\dot{G}}{G}$ is positive, implying that the gravitational constant is presently increasing with time. The condition for having $\frac{\dot{G}}{G} > 0$ is

$$t < t_0 + \frac{H_0 \varphi_0}{\rho_0} = 1.277 \times 10^{19} = 28.925\, t_0. \quad (14)$$

The gravitational constant will be decreasing with time and consequently $\frac{\dot{G}}{G}$ will be negative for $t \geq 28.925\, t_0$. It implies that beyond 28.925 times the present age of the universe, the gravitational constant will be decreasing with time. Using (2) and (5) we get,

$$\omega(\varphi) = -\frac{2}{3}\left(1 + \frac{\ddot{\varphi}\varphi}{\dot{\varphi}^2}\right) = -\frac{2}{9}\left(7 - \frac{\ddot{a}a}{\dot{a}^2}\right) = -\frac{2}{9}(7 + q). \tag{15}$$

Equation (15) shows that the Brans-Dicke parameter $\omega(\varphi)$ has a linear relationship with the deceleration parameter (q).

At $t = t_0$ we have,

$$\omega(\varphi_0) = -\frac{2}{9}(7 + q_0) = -1.341. \tag{16}$$

Substituting for q in equation (15) from equation (10)

$$\omega(\varphi) = -\frac{2}{9}(7 + q) = -\frac{2}{9}\left[6 + \frac{f\rho_0/\varphi_0}{\left[\frac{f\rho_0}{\varphi_0}(t_0 - t) + H_0\right]^2}\right]. \tag{17}$$

Equation (17) shows the time variation of Brans-Dicke parameter $\omega(\varphi)$.

Combining the equations (5) and (9) one gets,

$$\varphi = \varphi_0 a^{-3} = \varphi_0 Exp\left[-3\left(-\frac{1}{2}\frac{f\rho_0 t^2}{\varphi_0} + \left(H_0 + \frac{f\rho_0 t_0}{\varphi_0}\right)t - H_0 t_0 - \frac{1}{2}\frac{f\rho_0 {t_0}^2}{\varphi_0}\right)\right]. \tag{18}$$

Figure (5) shows the variation of the Brans-Dicke parameter $\omega(\varphi)$ as a function of the scalar field φ and figure (6) shows its time dependence. It is found to be negative over the entire range of study.

We have plotted the deceleration parameter and the gravitational constant as functions of the scale factor (a), in the figures (7) and (8) respectively.

Conclusions

In the present study we have assumed an empirical dependence of the BD scalar field parameter φ on the scale factor (a). This model clearly shows that a generalized scalar tensor theory, where the BD parameter ω is regarded as a function of the scalar field φ, can drive an accelerated expansion for the present universe. Here we have found that the universe has made a transition from a decelerated phase of expansion to the present accelerated phase and it will continue to remain in the state of acceleration. This study also shows that there was accelerated expansion in the very early stage of the universe before the beginning of the deceleration phase. These calculations reveal that the gravitational constant increases with time. The rate of this increase is consistent with other studies in this regard. The present study shows the variation of the BD parameter $\omega(\varphi)$ graphically as a function of time and also the scalar field parameter φ. To take into account the exchange of energy between the field of matter (both dark and baryonic) and dark energy we have introduced a function $f(t)$ in this model. It has been found to decrease with time indicating a slow conversion of matter into dark energy, which is considered to be responsible for this accelerated expansion of the universe.

FIGURES

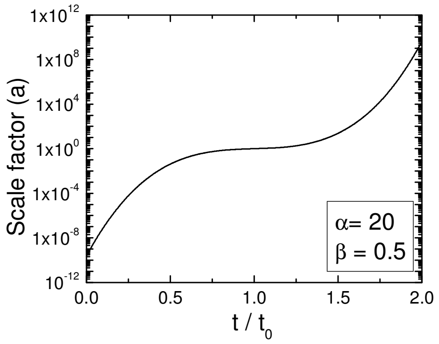

Figure 1. Variation of scale factor (a) as a function of time.

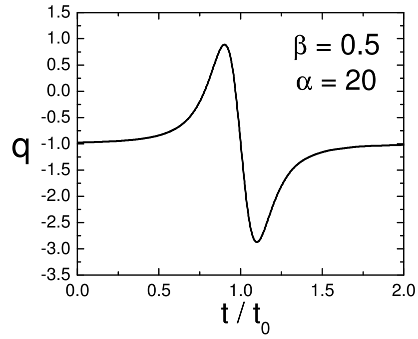

Figure 2. Variation of deceleration (q) parameter as a function of time.

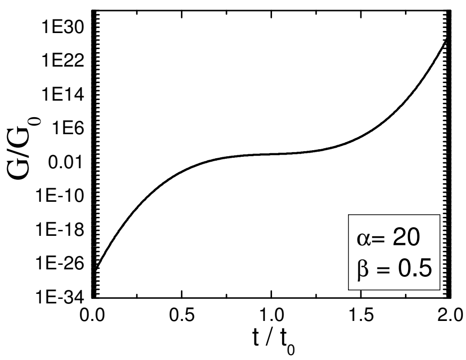

Figure 3. Variation of gravitational constant (G), as a function of time.

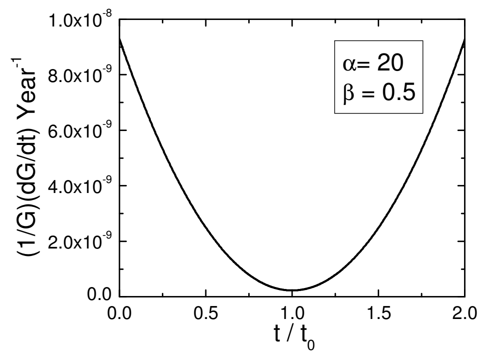

Figure 4. Variation of fractional change of G per second, as a function of time.

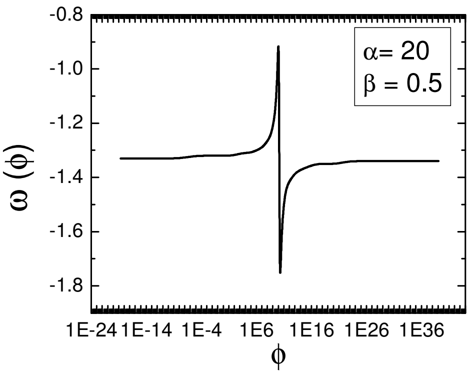

Figure 5. Variation of $\omega(\varphi)$ as a function of the scalar field φ.

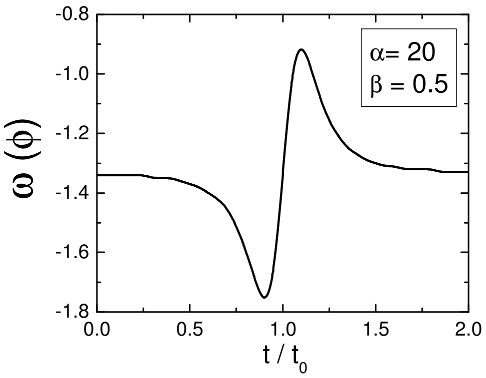

Figure 6. Variation of $\omega(\varphi)$ as a function of time.

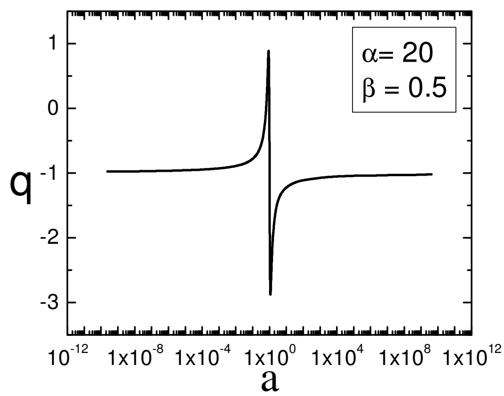

Figure 7. Variation of q as a function of scale factor (a).

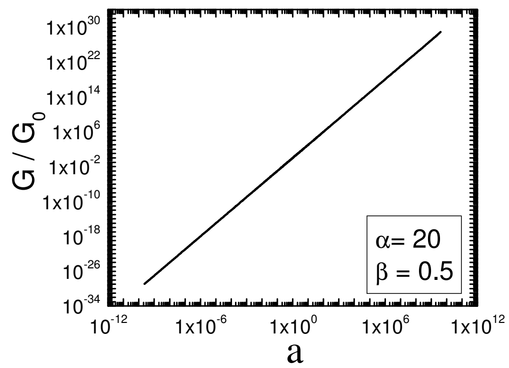

Figure 8. Variation of gravitational constant (G) as a function of the scale factor (a).

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