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RECURRENT FINSLER SPACES THEOREM IN PROJECTIVE RECURRENT SPACES OF THIRD ORDER

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Abstract: The present paper deals with study of recurrence of generalized second order in a Finsler space F_n^* equipped with non-symmetric connection. We have tried to make study of special birecurrent and special generalized birecurrent F_n^* of first and second kinds and have obtained results of significance. An attempt has also been made to study the recurrence of third order in such a Finsler space with special reference to the first curvature tensor $R^i_{jk\ell}(x,\dot{x})$ of F_n^*.

Keywords: Finsler spaces, special birecurrent, Ricci-recurrent spaces.

INTRODUCTION

The recurrent Finsler spaces have been studied by Mishra and Pande [3], Sen, R.N. [1], Chaki and Chaudhary [2], have introduced the Ricci-recurrent spaces of second order and studied the properties of the recurrence vector and tensor field and the curvature tensor field H^i_{jkh} in an n-dimensional Finsler space Fn equipped with Berwald's symmetric connection coefficient $G^i_{jk}(x,\dot{x})$. Ray, A. K. [4] has defined the generalized 2-recurrent Riemannian space. An attempt to extend the theory of generalised 2-recurrent curvature tensor to Finsler geometry has been made by Pande and Khan [5] and many more. Pande studied recurrent Finsler spaces of third order having symmetric connection parameter G^i_{jk} and Berwald's covariant derivative defined in it. Pandey and Mishra, R.B. [6] has carried out a comparative study of various types of recurrent Finsler spaces by using Berwald's and Cartan's curvature tensors.

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1. SPECIAL R^{+}- BIRECURRENT F_n^*:

Differentiating $\overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{\ell} = \lambda_\ell R^h_{ijk}$ $\oplus$ - covariantly with respect to x^m and thereafter using $\overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{\ell m} = \lambda_\ell \overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{m}$, we get

$$\textbf{(1.1)}\qquad \lambda_\ell\,{}^{+}\big|_{m} = 0 \qquad \text{or} \qquad \overset{+}{R}{}^h_{ijk} = 0.$$

Therefore, we can state:

THEOREM (1.1):

If a R^{+}-recurrent F_n^* of first order be special R^{+}-birecurrent of second kind then such an F_n^* is ether flat one or in such an F_n^* the recurrence vector is a covariant constant.

Let us now suppose that the special birecurrent F_n^* of first kind be also 1-recurrent then in such a case we can easily state the following:

COROLLARY (1.1):

If a special birecurrent F_n^* of first kind be also 1-recurrent then such an F_n^* is either flat one or in such an F_n^* the recurrence vector is a $\oplus$ - covariant constant.

Commutating $\overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{\ell m} = \lambda_m \overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{\ell}$ with respect to the indices ℓ and m and thereafter using the relevant commutation formula, we get

$$\textbf{(1.2)}\qquad \left(\dot{\partial}_r \overset{+}{R}{}^h_{ijk}\right) \overset{+}{R}{}^r_{pjk}\, \dot{x}^p + \overset{+}{R}{}^r_{ijk} \overset{+}{R}{}^h_{r\ell m} - \overset{+}{R}{}^h_{rjk} \overset{+}{R}{}^h_{i\ell m} - \overset{+}{R}{}^h_{irk} \overset{+}{R}{}^r_{j\ell m} -$$

$$- \overset{+}{R}{}^h_{ijr} \overset{+}{R}{}^r_{k\ell m} - \overset{+}{R}{}^h_{ijk}\,|_p\, N^p_{m\ell} = 0.$$

While deriving (1.2), we have taken into account that the space under consideration is also 1-recurrent. Exactly a similar result can be obtained if we start our discussion from $\overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{\ell m} = \lambda_\ell \overset{+}{R}{}^h_{ijk}\,{}^{+}\big|_{m}$ and proceed in a like manner. Therefore, we can state:

THEOREM (1.2):

In a special birecurrent F_n^* of first and second kinds, (1.2) always holds. Transvecting $\overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell m} = \lambda_m \overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell}$ and $\overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell m} = \lambda_\ell \overset{+}{R}{}^h_{ijk}\,{}^+\big|_{m}$ by the metric tensor g^{ip}, we get

(1.3) a) $\overset{+}{R}_{jpkh}\,{}^+\big|_{m}\,{}^+\big|_{\ell} = \lambda_\ell \overset{+}{R}_{jpkh}\,{}^+\big|_{m}$

And b) $\overset{+}{R}_{jpkh}\,{}^+\big|_{m}\,{}^+\big|_{\ell} = \lambda_m \overset{+}{R}_{jpkh}\,{}^+\big|_{\ell}$

respectively. Conversely the transvection of (1.3a) and (1.3b) by the associate tensor g^{ip} of the metric tensor g_{ij} yield $\overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell m} = \lambda_m \overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell}$ and $\overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell m} = \lambda_\ell \overset{+}{R}{}^h_{ijk}\,{}^+\big|_{m}$ respectively. Then, the above conditions are equivalent to the conditions (1.3a) and (1.3b) respectively. Therefore, we can state:

THEOREM (1.3):

A special R^+-birecurrent space of first kind and special R^+-birecurrent space of second kind may be characterized by the equation (1.3a) and (1.3b) respectively.

Contracting $\overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell m} = \lambda_m \overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell}$ and $\overset{+}{R}{}^h_{ijk}\,{}^+\big|_{\ell m} = \lambda_\ell \overset{+}{R}{}^h_{ijk}\,{}^+\big|_{m}$ with respect to the indices *i* and *h*, we get

(1.4) a) $\overset{+}{R}_{jk}\,{}^+\big|_{m}\,{}^+\big|_{\ell} = \lambda_\ell \overset{+}{R}_{jk}\,{}^+\big|_{m}$

And b) $\overset{+}{R}_{jk}\,{}^+\big|_{m}\,{}^+\big|_{\ell} = \lambda_m \overset{+}{R}_{jk}\,{}^+\big|_{\ell}$

respectively. Thus, the Ricci tensor R_{jk} of a special R^+-birecurrent F_n^* of first and second kinds satisfy (1.4a) and (1.4b) respectively. Conversely if the Ricci tensor of a Finsler space F_n^* satisfies (1.4a) or (1.4b) then the space need not be special R^+-birecurrent of first and second kinds respectively, Therefore, we can state:

THEOREM (1.4):

The Ricci tensor R_{jk} of a special R^+-birecurrent F_n^* of first and second kinds respectively satisfy (1.4a) and (1.4b) and conversely if the Ricci tensor of a Finsler space F_n^* satisfies

(1.4a) or (1.4b) then the space need not be special R^+-birecurrent of first and second kinds respectively. Transvecting (1.4a) and (1.4b) by g^{jp}, we get

(1.5) a) $$\overset{+}{R}{}_k^p \overset{+}{\big|}_m \overset{+}{\big|}_\ell = \lambda_\ell \overset{+}{R}{}_k^p \overset{+}{\big|}_m ,$$

And b) $$\overset{+}{R}{}_k^p \overset{+}{\big|}_m \overset{+}{\big|}_\ell = \lambda_m \overset{+}{R}{}_k^p \overset{+}{\big|}_\ell .$$

Contracting the indices p and k in (1.5a) and (1.5b) and thereafter using the fact that

$R_p^{+p} = R^+$, we get

(1.6) a) $$\overset{+}{R}{}^+ \big|_m \overset{+}{\big|}_\ell = \lambda_\ell \overset{+}{R}{}^+ \big|_m$$

and b) $$\overset{+}{R}{}^+ \big|_m \overset{+}{\big|}_\ell = \lambda_m \overset{+}{R}{}^+ \big|_\ell .$$ Therefore, we can state:

THEOREM (1.5):

In a special birecurrent F_n^* of first and second kinds the contracted tensor *R* also behaves accordingly.

2. GENERALISED 2-RECURRENT $R^+F_n^*$:

DEFINITION (2.1):

A Finsler space F_n^* is said to be generalized birecurrent of first kind if the curvature tensor $\overset{+}{R}{}_{ijk}^h$ satisfies the relation

(2.1) $$\overset{+}{R}{}_{ijk}^h \overset{+}{\big|}_{mn} = \beta_n \overset{+}{R}{}_{ijk}^h \overset{+}{\big|}_m + \alpha_{mn} \overset{+}{R}{}_{ijk}^h$$

where β_n and α_{mn} are respectively the non zero associated vector and associated tensor of recurrence.

We commute (2.1) with respect to the indices *m* and *n* and thereafter use the relevant commutation formula and get

(2.2) $$-\overset{+}{R}{}_{irijk}^h \overset{+}{R}{}_{mn}^r + \overset{+}{R}{}_{ijk}^r \overset{+}{R}{}_{rmn}^h - \overset{+}{R}{}_{rjk}^h \overset{+}{R}{}_{imn}^r - \overset{+}{R}{}_{irk}^h \overset{+}{R}{}_{jmn}^r + N_{nm}^r \left(\overset{+}{R}{}_{ijk}^h \big|_r \right) -$$

$$- \overset{+}{R}{}_{ijr}^h \overset{+}{R}{}_{kmn}^r = \beta_n \overset{+}{R}{}_{ijk}^h \overset{+}{\big|}_m - \beta_m \overset{+}{R}{}_{ijk}^h \overset{+}{\big|}_n + (a_{mn} - a_{nm}) \overset{+}{R}{}_{ijk}^h .$$

With the help of (2.2) we can easily verify that a_{mn} is non-symmetric however if we assume that a_{mn} is symmetric recurrence tensor and R^h_{ijk} is a first order recurrent curvature tensor with respect to given associated vector of recurrence then from (2.2), we get

$$\textbf{(2.3)}\quad -\overset{+}{R}{}^h_{rijk}\overset{+}{R}{}^r_{mn}+\overset{+}{R}{}^r_{ijk}\overset{+}{R}{}^h_{rmn}-\overset{+}{R}{}^h_{rjk}\overset{+}{R}{}^n_{imn}-\overset{+}{R}{}^h_{irk}\overset{+}{R}{}^r_{jmn}-\overset{+}{R}{}^h_{ijr}\overset{+}{R}{}^n_{kmn}+ +\beta_r\overset{+}{R}{}^h_{ijk}N^r_{nm}=0.$$

Contracting (2.3) with respect to the indices *h* and *k* and using the relevant commutation formula $X^i\,{}^+|_{jk}-X^i\,{}^+|_{kj}=-\left(\dot\partial_m X^i\right)R^m_{pjk}\dot x^p+X^m R^i_{mjk}+X^i\,{}^+|_m\,N^m_{kj}$, we get

$$\textbf{(2.4)}\quad -\left(\dot\partial_r\overset{+}{R}_{ij}\right)\overset{+}{R}{}^r_{mn}-\overset{+}{R}_{rj}\overset{+}{R}{}^r_{imn}-\overset{+}{R}_{ir}\overset{+}{R}{}^r_{jmn}+\beta_r\overset{+}{R}_{ij}N^r_{nm}=0.$$

The following identities can be easily obtained

$$\textbf{(2.5)}\quad \text{a)}\quad \overset{+}{R}_k=R^+_{jk}\dot x^j,\qquad \text{b)}\quad \overset{+}{R}_{ij}\dot x^i\dot x^j=\overset{+}{R}_j\dot x^j=(n-1)\overset{+}{R}$$

$$\text{c)}\quad \left(\dot\partial_r\overset{+}{R}_j\right)\dot x^j=(n-1)\dot\partial_r\overset{+}{R}-\overset{+}{R}_r.$$

Transvecting (2.4) successively by $\dot x^i$ and $\dot x^j$ and thereafter using (2.5a, b, c), we get

$$\textbf{(2.6)}\quad \left(\dot\partial_r\overset{+}{R}\right)\overset{+}{R}{}^r_{mn}=\beta_r\overset{+}{R}N^r_{nm}.$$ Therefore, we can state:

THEOREM (2.1):

In a R^+ generalized 2-F^*_n (2.6) always holds. Transvecting (2.1) by $\dot x^i$, we get

$$\textbf{(2.7)}\quad \overset{+}{R}{}^h_{jk}\,{}^+\big|_{mn}=\beta_n\overset{+}{R}{}^h_{jk}\,{}^+\big|_m+a_{mn}\overset{+}{R}{}^h_{jk}.$$

Commutating (2.7) with respect to the indices *m* and *n* and using the relevant commutation formula, we get

$$\textbf{(2.8)}\quad -\left(\dot\partial_r R^h_{jk}\right)\overset{+}{R}{}^r_{mn}+\overset{+}{R}{}^r_{jk}\overset{+}{R}{}^h_{rmn}-\overset{+}{R}{}^h_{rk}\overset{+}{R}{}^r_{jmn}-\overset{+}{R}{}^h_{jr}\overset{+}{R}{}^h_{kmn}+ +\left(\overset{+}{R}{}^h_{jk}\,{}^+\big|_r\right)N^r_{nm}=\beta_n\overset{+}{R}{}^h_{jk}\,{}^+\big|_m-\beta_m\overset{+}{R}{}^h_{jk}\,{}^+\big|_n+\left(\alpha_{mn}-\alpha_{nm}\right)\overset{+}{R}{}^h_{jk}.$$

Differentiating (2.8) ⊕ - covariantly with respect to x^ℓ and then transvecting the result thus obtained, we get

(2.9)
$$(\alpha_{mn}-\alpha_{nm})^{+}\big|_{\ell}\overset{+}{R}{}^{h}_{k}+(\alpha_{mn}-\alpha_{nm})\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{\ell}+\beta_{n}\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{m\ell}+$$
$$+\beta_{n}{}^{+}\big|_{\ell}\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{m}-\beta_{m}\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{n\ell}-\beta_{m}{}^{+}\big|_{\ell}\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{n}$$
$$=-\left(\dot{\partial}_{r}\overset{+}{R}{}^{h}_{jk}\right)^{+}\big|_{\ell}\dot{x}^{r}\overset{+}{R}{}^{r}_{mn}-\left(\dot{\partial}_{r}\overset{+}{R}{}^{h}_{jk}\right)\dot{x}^{j}\overset{+}{R}{}^{r}_{mn}{}^{+}\big|_{\ell}+\overset{+}{R}{}^{h}_{rmn}\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{\ell}+$$
$$+\overset{+}{R}{}^{r}_{k}\overset{+}{R}{}^{h}_{rmn}{}^{+}\big|_{\ell}-\overset{+}{R}{}^{h}_{rk}{}^{+}\big|_{\ell}\overset{+}{R}{}^{r}_{mn}-\overset{+}{R}{}^{h}_{rk}\overset{+}{R}{}^{r}_{mn}{}^{+}\big|_{\ell}-\overset{+}{R}{}^{h}_{r}{}^{+}\big|_{\ell}\overset{+}{R}{}^{r}_{kmn}-$$
$$-\overset{+}{R}{}^{h}_{r}\overset{+}{R}{}^{r}_{kmn}{}^{+}\big|_{\ell}+\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{r\ell}N^{r}_{nm}+\overset{+}{R}{}^{h}_{k}{}^{+}\big|_{r}N^{r}_{nm}{}^{+}\big|_{\ell}.$$

Contracting (2.9) with respect to the indices h and k and then using (2.1) and the set of equations given by (2.5), we get

(2.10)
$$R^{+}\big|_{r}\left(\beta_{\ell}N^{r}_{nm}+N^{r}_{nm}{}^{+}\big|_{\ell}\right)+\alpha_{r\ell}\overset{+}{R}N^{r}_{nm}$$
$$=\Big[(n-1)(\alpha_{mn}-\alpha_{nm})^{+}\big|_{\ell}+(\beta_{n}\alpha_{m\ell}-\beta_{m}\alpha_{n\ell})\overset{+}{R}+$$
$$+(\alpha_{mn}-\alpha_{nm})\overset{+}{R}\big|_{\ell}+\left(\beta_{n}\beta_{\ell}+\beta_{n}{}^{+}\big|_{\ell}\right)R^{+}\big|_{m}-$$
$$-\left(\beta_{m}\beta_{\ell}+\beta_{m}{}^{+}\big|_{\ell}\right)R^{+}\big|_{n}+\left\{\left(\dot{\partial}_{r}\overset{+}{R}\right)\overset{+}{R}{}^{r}_{mn}\right\}^{+}\big|_{\ell}\Big].$$

Allowing a cyclic interchange of the indices ℓ, m and n in (2.10) and thereafter adding all the three equations thus obtained, get

(2.11)
$$\left(R^{+}\big|_{r}\beta_{[\ell}+\overset{+}{R}\alpha_{r[\ell}\right)N^{r}_{nm]}+\overset{+}{R}\big|_{r}N^{r}_{[nm}{}^{+}\big|_{\ell]}$$
$$=(n-1)\Big[\left\{\gamma_{[mn}{}^{+}|_{\ell]}+\beta_{[n}\gamma_{m\ell]}\right\}\overset{+}{R}+\gamma_{[mn}R^{+}|_{\ell]}+\beta_{[n}{}^{+}|_{\ell}\overset{+}{R}_{m]}-$$
$$-\beta_{[m}{}^{+}|_{\ell}R^{+}|_{n]}+\left\{\left(\dot{\partial}_{r}\overset{+}{R}\right)\overset{+}{R}{}^{r}_{[mn}\right\}^{+}\big|_{\ell}\Big],$$

where,

(2.12) $\gamma_{mn} \overset{dif.}{=} \alpha_{mn} - \alpha_{nm}.$ Therefore, we can state:

THEOREM (2.2):

In a Finsler space F_n^* with generalized birecurrent curvature tensor $\overset{+}{R}{}^h_{ijk}$, the identity (2.11) always holds. Differentiating (2.7) partially with respect to $\dot{x}^i$ we get

$$\textbf{(2.13)}\ \dot{\partial}_i\left(\overset{+}{R}{}^h_{jk}\,{}^{+}\big|_{mn}\right) = \dot{\partial}_i\left(\overset{+}{R}{}^h_{jk}\big|_m\right)\beta_n + \overset{+}{R}{}^h_{jk}\,{}^{+}\big|_m\left(\dot{\partial}_i\beta_n\right) + \left(\dot{\partial}_i\alpha_{mn}\right)\overset{+}{R}{}^h_{jk} + \alpha_{mn}\left(\dot{\partial}_i\overset{+}{R}{}^h_{jk}\right).$$

Using (a) $\dot{x}^i\,{}^{+}|_k = 0 = \dot{x}^i\,{}^{-}|_k$, (b) $R^i_{jk} = R^i_{hjk}\dot{x}^h$, (c) $R^i_j = R^i_{hj}\dot{x}^h$,

(d) $R^i_{hjk} = -R^i_{hkj}$, $R^i_{jk} = -R^i_{kj}$, (e) $N^i_{jk} = -N^i_{kj} = \Gamma^i_{jk} - \Gamma^i_{kj}$, (f) $\Gamma^i_{hjk} = \dot{\partial}_h\Gamma^i_{jk}$.

And (2.1) the identity (2.13) assumes the following form

$$\begin{aligned}
&\textbf{(2.14)}\ \dot{x}^q\overset{+}{R}{}^h_{iqjk}\,{}^{+}\big|_{mn} + \Big\{\overset{+}{R}{}^r_{jk}\,{}^{+}\big|_n\Gamma^h_{irm} + \overset{+}{R}{}^r_{jk}\,{}^{+}\big|_m\Gamma^h_{irn} - \overset{+}{R}{}^h_{rk}\,{}^{+}\big|_n\Gamma^r_{ijm} - \\
&- \overset{+}{R}{}^h_{rk}\,{}^{+}\big|_m\Gamma^r_{ijn} - \overset{+}{R}{}^h_{jr}\,{}^{+}\big|_n\Gamma^r_{ikm} - \overset{+}{R}{}^h_{jr}\,{}^{+}\big|_m\Gamma^r_{ikn} - \Big(\overset{+}{R}{}^h_{rjk}\,{}^{+}\big|_n + \\
&+ \dot{x}^q\overset{+}{R}{}^r_{rqjk}\,{}^{+}\big|_n\Big)\Gamma^r_{ipm}\dot{x}^p - \Big(\overset{+}{R}{}^h_{rjk}\,{}^{+}\big|_m + \dot{x}^q\overset{+}{R}{}^h_{rqjk}\,{}^{+}\big|_m\Big)\Gamma^r_{ipn}\dot{x}^p\Big\} + \\
&+ \overset{+}{R}{}^r_{jk}\Gamma^h_{irm}\,{}^{+}\big|_n - \overset{+}{R}{}^h_{rk}\Gamma^r_{ijm}\,{}^{+}\big|_n - \overset{+}{R}{}^h_{jr}\Gamma^r_{ikm}\,{}^{+}\big|_n\Big(\overset{+}{R}{}^h_{rjk} + \\
&+ \dot{x}^q\overset{+}{R}{}^h_{rqjk}\Big)\Gamma^r_{ipm}\,{}^{+}\big|_n\dot{x}^p - \overset{+}{R}{}^h_{jk}\,{}^{+}\big|_n\Gamma^r_{imn} - \Big\{\overset{+}{R}{}^s_{jk}\Gamma^h_{rsm} - \\
&- \overset{+}{R}{}^h_{sk}\Gamma^s_{rjm} - \overset{+}{R}{}^h_{js}\Gamma^s_{rkm} - \Big(\overset{+}{R}{}^h_{sjk} + \dot{x}^q\overset{+}{R}{}^h_{sqjk}\Big)\Gamma^s_{rtm}\dot{x}^t\Big\}\Gamma^r_{ipn}\dot{x}^p \\
&= \beta_n\Big\{\dot{x}^q\overset{+}{R}{}^h_{iqjk}\,{}^{+}\big|_m + \overset{+}{R}{}^r_{jk}\Gamma^h_{irm} - \overset{+}{R}{}^h_{rk}\Gamma^r_{ijm} - \overset{+}{R}{}^h_{jr}\Gamma^r_{ikm} -
\end{aligned}$$

$$-\left(\overset{+}{R}{}^{h}_{rjk}+\dot{x}^{q}\overset{+}{R}{}^{h}_{rqjk}\right)\Gamma^{r}_{ipm}\dot{x}^{p}\Bigg\}+\overset{+}{R}{}^{h}_{jk}\,{}^{+}\Big|_{m}\left(\dot{\partial}_{\ell}\beta_{n}\right)+$$

$$+\left(\dot{\partial}_{\ell}\alpha_{mn}\right)\overset{+}{R}{}^{h}_{jk}+\alpha_{mn}\dot{x}^{q}\overset{+}{R}{}^{h}_{iqjk}\,.$$

Commutating (2.14) with respect to the indices m and n and then writing $\Gamma^{h}_{ijk}\,{}^{+}\big|_{m}=\beta_{n}\Gamma^{h}_{ijk}$, we get

(2.15) $$\dot{x}^{q}\Bigg\{\overset{+}{R}{}^{h}_{iqjk}\,{}^{+}\Big|_{[mn]}-\overset{+}{R}{}^{h}_{sqjh}\,\dot{x}^{t}\Gamma^{s}_{rt[m}\Gamma^{r}_{<ip>n]}\dot{x}^{p}-\overset{+}{R}{}^{h}_{iqjh}\,{}^{+}\Big|_{[m}\beta_{n]}-$$

$$-\overset{+}{R}{}^{h}_{iqjh}\,\alpha_{[mn]}\Bigg\}$$

$$=\frac{1}{2}R_{j}\,{}^{+}\Big|_{r}\dot{\partial}_{i}N^{r}_{mn}+\left\{\overset{+}{R}_{s}\Gamma^{s}_{rj[m}+\overset{+}{R}_{s}\,\dot{x}^{t}\Gamma^{s}_{rt[m}\right\}\Gamma^{r}_{<ip>n]}\dot{x}^{p}+$$

$$+\overset{+}{R}_{j}\,\dot{\partial}_{i}\alpha_{[mn]}+\overset{+}{R}_{j}\,{}^{+}\Big|_{[m}\dot{\partial}_{<i>}\beta_{n]}.$$

Therefore, we can state:

THEOREM (2.3):

In a Finsler space F_n^* with generalized birecurrent curvature tensor R^h_{ijk} the identity given by (2.15) always holds provided Γ^h_{ijk} be supposed to be first order recurrent with respect to the associated vector of recurrence.

DEFINITION (2.2):

A Finsler space F_n^* is said to be generalized birecurrent of second kind if the curvature tensor R^h_{ijk} satisfies the following relation

(2.16) $$\overset{+}{R}{}^{h}_{ijk}\,{}^{+}\Big|_{mn}=\beta_{m}\overset{+}{R}{}^{h}_{ijk}\,{}^{+}\Big|_{n}+\alpha_{mn}\overset{+}{R}{}^{h}_{ijk}$$

where β_m and α_{mn} are respectively the non-zero associated vector and associated tensor of recurrence.

According to the provisions of this definition, if we carry out calculations as have been carried out in the foregoing lines of this section, we can easily arrive at the following conclusions, which are being described in the form of corollaries.

COROLLARY (2.1): In a R^{+} generalized birecurrent space the following always holds

(2.17) $$\left(\dot{\partial}_r \overset{+}{R}\right)\overset{+}{R}{}^{r}_{nm} - \beta_r \overset{+}{R} N^{r}_{mn} = 0 .$$

COROLLARY (2.2):

In a R^{+}- generalized birecurrent space of the second kind the following identity always holds

(2.18) $$\left(R^{+}\Big|_{r} \beta_{[\ell} + \overset{+}{R}\alpha_{r[\ell}\right)N^{r}_{mn]} + R^{+}\Big|_{r} N^{r}_{[mn}{}^{+}\Big|_{\ell]}$$

$$\beta_{[m}{}^{+}\Big|_{\ell} R^{+}\Big|_{n]} - \beta_{[n}{}^{+}\Big|_{\ell} R^{+}\Big|_{m]} + \left\{\left(\dot{\partial}_r R\right)\overset{+}{R}{}^{r}_{[nm}\right\}^{+}\Big|_{\ell]}\Bigg] .$$

COROLLARY (2.3):

In a R^{+}- generalized birecurrent space of the second kind the identity analogous to (4.15) is given by

(2.19) $$\dot{x}^{q}\left\{\overset{+}{R}{}^{h}_{iqjk}{}^{+}\Big|_{[nm]} - \overset{+}{R}{}^{h}_{sqjk}\,\dot{x}^{t}\Gamma^{s}_{rt[n}\Gamma^{r}_{<ip>m]}\dot{x}^{p} - \overset{+}{R}{}^{h}_{iqjh}{}^{+}\Big|_{n]}\beta_{m]} - -\overset{+}{R}{}^{h}_{iqjk}\,\alpha_{[nm]}\right\}$$

$$= \frac{1}{2} R_{j}{}^{+}\Big|_{r}\dot{\partial}_{i} N^{r}_{nm} + \left\{\overset{+}{R}{}^{h}_{s}\,\Gamma^{s}_{rj[n} + \overset{+}{R}{}^{h}_{sj}\,\dot{x}^{t}\Gamma^{s}_{rt[n}\right\}\Gamma^{r}_{<ip>m]}\dot{x}^{p} +$$

$$+\overset{+}{R}_{j}\,\dot{\partial}_{i}\alpha_{[mn]} + \overset{+}{R}_{j}{}^{+}\Big|_{[n}\dot{\partial}_{<i>}\beta_{m]} .$$

Provided Γ^{h}_{ijk} be supposed to be first order recurrent with respect to associated vector of recurrence.

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