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NUMERICAL SIMULATION USING ARTIFICIAL BEE COLONY ALGORITHM WITH RBF

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Abstract: The numerical analysis of a partial differential equation using radial basis function requires the value of the shape parameter that defines the shapes of RBFs. In the presented work, an innovative scheme is proposed to evaluate the numerical solutions of the Fitzhugh-Nagumo equation using one of the optimization techniques named Artificial Bee Colony algorithm integrated with RBF method for optimizing the shape parameter. Optimization is based on the concept of enhancing the positive characteristics of a mathematical model with diminishing negative aspects. It has been useful in determining the parameters required to solve mathematical models in a variety of ways. This approach is used with RBF Pseudo-spectral method which is mesh free technique for numerical simulation of partial differential equations. The obtained numerical solutions for the three considered examples highlight the efficacy of this method. The results are derived by using the Guassian RBF, multi-qudric RBF, and Cubic matern RBF as basis functions for numerical approximations which are in good agreement.

Keywords: Optimization, Fitzhugh-Nagumo Equation, Radial Basis Function, Pseudo spectral method.

1. Introduction

In mathematical research, non-linear PDEs are intensively practiced to emulate the physical phenomena of the natural model. Numerical solutions of PDEs provide valuable facts and an improved form of mathematical models. The Fitzhugh Nagumo (FNE) equation is a type of non- linear partial differential equation presented as:

$$u_t = u_{xx} + u(u-\alpha)(1-u), \quad 0 < \zeta < 1 \tag{1}$$

Here, $u(x,t)$ is an anonymous factor depends upon the arbitrary constant. Nagumo and Fitzhugh proposed the FN equation for solving the problematic models of nerve axon and membrane [1-2]. The presented equation has various real life applications such as in population genetics, autocatalytic chemical reactions, circuit theory, and neurophysiology [3-5].

In this paper, the Fitzhugh-Nagumo equation is solved numerically using the hybrid approach of one of the optimization algorithms named Artificial Bee Colony (ABC) and RBFs. The

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associated shape parameter of RBFs is found out by the proposed approach of ABC with one of the RBF pseudo spectral method with minimizing the errors. RBF-PS process is an inventive scheme to find the solutions of various PDEs numerically.

1.1. Importance of Radial Basis Function (RBF)

The RBF pseudo-spectral technique does not require any meshes. Numerical treatment of various ordinary and partial differential equations requires a successful basis function named RBF. Approaches based on RBF are extremely novel and fruitful method for various mathematical problems based on geometrical complex domain. Hardy [6] firstly invented the approach based upon multiquadric RBF for interpolation purposes with topological quadric surface. Micchelli [7] made a step forward by demonstrating that multi-quadraic surface interpolation is always solvable. The MQ approach has the benefit of obtaining the interpolant using a linear combination of basis functions that are only dependent on the distance from a specific node, which is known as the center. In order to solve a PDE, Edward Kansa invented the Kansa method [8] in 1990. It first used the multi quadraic, a widely supported interpolant to analyze the behaviour of PDEs. The numerical simulation of a PDE using RBF requires the value of the shape parameter that evaluates the shapes of RBFs and needs to be evaluated precisely. RBFs offer flexibility and efficiency over the complex domain for solving various PDEs. For detailed explanation of RBF with their applications refer the published work in 2023[9]. In 2023, RBF-PS method employed to analyze the solutions of PDE with optimization technique [10] by optimizing the parameter of RBF.

1.2. Significance of Optimization Techniques

The present novel approach is based on one of the optimization techniques which are used by the researchers to find the optimum feasible solution to a problematic natural model. These techniques are applied in various fields to find the solution to mathematical models or physical phenomena that best exploits or minimizes their parameters. For example, minimizing the cost factor in the production of goods or services and maximizing profit are common needs that require optimization tools. These optimization techniques like genetic algorithms (GA), particle swarm optimization (PSO), artificial bee colony algorithm (ABC), ant colony algorithm (ACO), differential evolution (DE), and bacteria foraging optimization (BFO) etc. can be customized for a particular class of problems based upon the concept of Swarm intelligence (SI). Refer the publications for the applications of optimization techniques in solving mathematical problems numerically [11].

Firstly, Artificial Bee Colony (ABC) algorithm recommended by Karaboga [12] for solving numerical optimization problems in 2005 established upon the concept of honey bees. In this search intelligence (SI) theory, explorations for wholesome food sources are done by bees by hovering around the nest. In the study of Bonabeau et al. [13], behaviour of social insects is observed for creating the optimization algorithm or problem solving tools. Algorithms based on the concept of SI become very productive to find the solutions of various mathematical models. In [14], a modified form of ABC named Adaptive ABC (A-ABC) is employed for analysing the current situation of transport energy demand (TED). A scheme based on different optimization algorithms is proposed by authors of [15] for figuring out Turkey’s condition about natural gas demand forecast grounded by meteorological indicator and also modified artificial bee colony (M-ABC) based approach [16] is projected for Turkey's energy convention. There are various other applications using different optimization techniques in various fields including numerical analysis of mathematical models.

The layout of the article is as follows: Section 2 represents the detailed methodology including the ABC technique with proposed algorithm and the numerical approach of the implementation of the RBF pseudo spectral process to find the solutions of FN equation. In Section 3, three test problems of the Fitzhugh-Nagumo equation are discussed with their numerical results using Cubic matren (CU) RBF, Multi-quadric (MQ) RBF and Gaussian RBF at different node points by applying the proposed hybrid approach. Section 4 concludes the outcomes of the proposed method.

2. Methodology with Detailed ABC Algorithm

2.1. Artificial Bee Colony (ABC) Algorithm

ABC optimization technique originated by Karaboga [13] in 2005 and this technique is based on the concept of swarm intelligence to simulate the foraging behaviour of honeybees. ABC has the advantages of being reliable, efficient, and simple to implement. In this algorithm, particles are in the form of bees like employed bees (workers), onlooker bees (observers), and scout bees. The ABC algorithm starts with the deployment of bees and their dispersion in various regions. Bees that are actively searching for nectar are all employed bees which convey the facts about the food to other bees. Some observer bees are waiting for the scout bees for finding the information about the fresh food sources. Employed bees dance in a designated area to exchange information about the food availability. Observer bees keep an eye on the dance and decide which food source to use based on its reliability. Better food sources are selected by more bees. All working bees turn into scouts when a food source is depleted.

Each food source in this technique represents a potential solution to the problematic model, and the amount of nectar represents the assessment of the fitness value. There is exactly one busy bee for every potential food source, and the entire number of food sources is equal to the total number of bees at work. An observer bee chooses a food source based on the associated probability factor defined below.

$$P_j = \frac{Fit_j}{\sum_{t=1}^{Np} Fit_t} \quad (2)$$

Here, the employed bees are relative to the ideal solution that evaluates the fitness Fit_j parameter of the j^{th} solution, and N_p represents the number of food sources. Fitness of a solution is derived for $f \geq 0$ as $\frac{1}{1+f}$ and for $f < 0$ as $1 + |f|$ where f is objective function value. The observer bee uses (3) to travel to new spot with the assistance of the selected employed bee.

$$v_j = x_j + \emptyset_j(x_j - x_k) \quad (3)$$

Here, the current position is indicated byx_j, selection of the employed bees is indicated byx_k, and $\emptyset_j$ is chosen at randomly -1 to 1 to locate the best food sources. After several iterations, every bee that is incapable of discovering a superior food source gets replaced by a scout bee. Using (4), the scout bees sent to substitute for the failed bee hovers around any random or unexplored location to explore its surroundings.

$$x_t = Lb + \emptyset_j(UB - LB) \quad (4)$$

Here UB and LB represents the upper bound and the lower bound, respectively. A position that cannot be improved during the specified cycles is utilized to determine the parameter limit (number of cycles). The three phases of ABC are shown in the next section with their pseudo codes.

a. Employed Phase

The following considerations should be kept in mind for the production of new solutions during this stage:

1. For the formulation of a new solution, modification of a randomly chosen variable is essential.

$$x^j_{new} = x^j + \vartheta(x^j - x^j_p)$$

Where x^j_p = j^{th} variable of p^{th} solution

x^j_{new} = j^{th} variable of new solution

$\vartheta =$ Random variable lies between -1 to 1

$x^j = j^{th}$ variable of current solution

2. Bounds of generated solution.

$x_{new}^{j} = lb$; if $x_{new}^{j} < lb$

$x_{new}^{j} = ub$; $if\ x_{new}^{j} > ub$

Here, evaluates the value of the objective function and determine the fitness value of the new solution using the conditions of bounds. Choose the newly generated solution to update the current one. Keep track of trial of every solution's failures. Reset it if the new solution is better; if not then increase the trial by one. The updated solution is considered for further process.

b. Onlooker Bees Phase

In this phase, bees in the waiting area of hive acquired the information from employed bees then onlooker bees select the food sources based on probability. There is a condition of probability (a parameter) to exploiting a specific food source. As by the fitness of each food source calculate the probability determined by (2).

c. Scout Bees Phase

In this phase bees hover around the hive and select the food randomly. There is a constraint to enter into the present phase for which parameter limit (user defined integer) is required. The parameter trial is connected with every solution and cost of trial should be greater than the value of limit. Initialize the trial of unrestrained solution to be 0. Each solution does not pass through this phase. The formula Np*d is applied for extracting the value of limit where d is dimension. This phase can occurred only when trial counter of atleast one solution is greater than the limit value.

Hence, ABC starts with the initialization of bees and the process of ABC iteratively improves the solutions through the exploration of the employed bee, selection of the onlooker bee, and scout bees' replacement of stagnant solutions. The best solution obtained by the optimization process represents the solution to the mathematical problem. The ABC algorithm involves the bees exploring the search space for finding the best solutions which minimizes the errors associated to the problem.

2.2. Implementation of the RBF-PS Method

In this section, the method RBF-pseudo-spectral is projected in the simulation of the FN-equation (1) with thefollowing initial and boundary conditions:

$$u(x,0) = f(x), x \in [a,b], \tag{5}$$

$$u(a,t) = g_1(t), \; u(b,t) = g_2(t), t \in [0,T] \tag{6}$$

For solving the proposed equation using the RBF pseudo-spectral method, *i.e.*, an accurate and efficient procedure, two stages are required. In the first stage, equation (1) is converted into ordinary differential equations (ODEs) to approximate the derivatives with RBFs, and then the converted equations are solved using MATLAB. In the method RBF-PS, the boundary conditions are satisfied by the basis function, or we have to set them uniquely; these are the ways of forming the boundary conditions of the equation.

By discretizing the domain with N node points, it is possible to write the RBF approximation as follows due to the property of the linear combination of RBF:

$$u_n(x,t) = \sum_{j=1}^{N} \delta_j(t)\, \emptyset(\|x - x_j\|)$$

where the interpolation coefficients are $\gamma j(t)$ and the RBF is denoted by $\emptyset$. Based on the nature of the mathematical problem, different type of RBFs can be applied. In this study, the Cubic Matern function has been used**.** This equation can be transformed by the interpolation technique as

$$u_n(x_i,t) = \sum_{j=1}^{N} \delta_j(t)\, \emptyset(\|x_i - x_j\|),\; 1 \le x_i \le N$$

The representation of the above equation can be in the following matrix form

$$u_N = AY$$

Here, interpolation coefficients are $Y = [\delta_{1,}\delta_2, \delta_3, \dots \delta_n]^t$ and $A = [A_{ij}]$ is an evaluation matrix with elements$A_{ij} = \emptyset(\|x_i - x_j\|)$. Derivative of u_n is derived by the differentiation of the basis function as follows:

$$\frac{d}{dx} u_n(x_i,t) = \sum_{j=1}^{N} \delta_j(t) \frac{d}{dx} \emptyset(\|x_i - x_j\|)$$

The matrix representation of this equation with all node points of the domain can be as

$$u_N{}' = A_x Y' \quad \text{where} A_x = \frac{d}{dx} \emptyset(\|x_i - x_j\|).$$

It is essential for the assurance of the inverse of the interpolation matrix before using the RBF-PS approach. It is based on the selection of RBF and the node points. The positive definite RBF results the invertible interpolation matrix. By inverting the interpolation matrix, we can calculate

$$A^{-1} u_N = Y'$$

By putting interpolation coefficients, we get first derivative as

$$u_N{}' = A_x A^{-1} u_N$$

Similarly, we can evaluate second derivative as

$u_N'' = A_{xx}A^{-1}u_N$

Using all these results, problem equation (1) transform into following expression using scattered nodes $x_j, 1 \le j \le N$.

$\frac{du_N}{dx} - A_{xx}A^{-1}u_N = u_N(u_N - \zeta)(1 - u_N)$

The way to deal with boundary constraints is to find a suitable basis function or module for the conditions by imposing some other equations. Hence, the boundary conditions can be applied by changing the right side of the above equation and then obtaining the equation using an ODE solver. In this study, the system of ODEs is solved in MATLAB with ODE 45.

3. Numerical Applications with Results and Discussion

This part examines the numerical applications of the proposed approach using the RBF pseudo-spectral method, which divides the numerical simulation into two parts. In the first part, the approximated derivatives are determined using RBF, and in the second part, results are derived for the obtained equation with MATLAB using the PSO algorithm. In this work, the multi-quadric (MQ) RBF, cubic matern radial basis function (CMRBF) and Gaussian (GA) RBF are considered as basis functions for numerical approximations and then compared their results for i= 1 to N.

Absolute error $= \left|u_{exact}(x_{i,}t) - u(x_{i,}t)\right|$; $L_\infty = \max\left(\left|u_{exact}(x_{i,}t) - u(x_{i,}t)\right|\right)$;

$L_2 = \sqrt{h\sum_{i=1}^{N}\left|u_{exact}(x_{i,}t) - u(x_{i,}t)\right|^2}$ $L_{rms} = \sqrt{\frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{N}}\left|u_{exact}(x_{i,}t) - u(x_{i,}t)\right|^2}$;

Problem 1: Consider Newell-whitehead equation *i.e.*, special type of Fitzhugh- Nagumo equation (1) with α =−1 with exact solution [17] as $u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-1}{2\sqrt{2}}\left(x - \frac{3}{\sqrt{2}}t\right)\right)$.

Initial and boundary conditions are derived from exact solution. For problem 1, Table 1 represents error norms based on shape parameter with respective computational period at different time intervals as 0.001, 0.002 and 0.003 and obtained results are computed with CMRBF at 21 node points. The computed errors in table 2 and table 3 are calculated with MQ RBF and Gaussian RBF respectively based on same parameters at different time intervals. The generalisation based on derived errors has been prepared that are in good accuracy with MQ RBF. The respective optimizing value of parameter is 0.351522. The numerical representation of the problem 1 at N=21 are presented graphically by in figure 1.

Table1. $\boldsymbol{L_\infty}, \boldsymbol{L_2}$ and $\boldsymbol{L_{RMS}}$ errors for problem 1 with $\boldsymbol{\alpha} = -1$ and N=21, Δt=0.0001 with CMRBF, iteration 10

T	$\boldsymbol{L_\infty}$	$\boldsymbol{L_2}$	$\boldsymbol{L_{RMS}}$	**Shape Parameter**	**Elapsed Time (sec.)**
0.001	5.8483e-08	1.2762e-08	2.7849e-09	0.483295	0.012579
0.002	1.1643e-07	2.5407e-08	5.5444e-09	0.460448	0.012856
0.003	3.1079e-07	6.7821e-08	1.4800e-08	0.330524	0.013943

Table2.$\boldsymbol{L_\infty}, \boldsymbol{L_2}$ and $\boldsymbol{L_{RMS}}$ errors for problem 1 with $\boldsymbol{\alpha} = -1$ and N=21, Δt=0.0001 with MQ, iteration 10

T	$\boldsymbol{L_\infty}$	$\boldsymbol{L_2}$	$\boldsymbol{L_{RMS}}$	**Shape Parameter**	**Elapsed time (sec.)**
0.001	1.0565e-09	2.3055e-10	5.0310e-11	0.351934	0.012015
0.002	2.9282e-09	6.3899e-10	1.3944e-10	0.351522	0.012896
0.003	3.3807e-08	7.3773e-09	1.6099e-09	0.554447	0.012903

Table3.$\boldsymbol{L_\infty}, \boldsymbol{L_2}$ and $\boldsymbol{L_{RMS}}$ errors for problem 1 with $\boldsymbol{\alpha} = -1$ and N=21, Δt=0.0001 with Gaussian RBF, iteration 10

T	$\boldsymbol{L_\infty}$	$\boldsymbol{L_2}$	$\boldsymbol{L_{RMS}}$	**Shape Parameter**	**Elapsed time (sec.)**
0.001	2.7675e-08	6.0391e-09	1.3178e-09	0.422531	0.011749
0.002	1.5841e-08	3.4569e-09	7.5435e-10	0.570723	0.011921
0.003	1.2848e-08	2.8037e-09	6.1182e-10	0.582431	0.012207

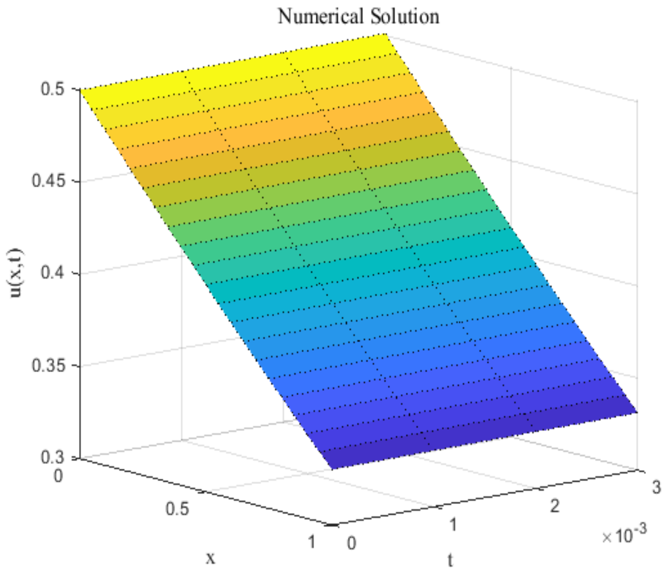

Figure 1. Graphical representation of solutions of problem 1 with N=21

Problem 2: Consider a non-linear general Fitzhugh-Nagumo equation (1) of dimension one in the domain [-22, 22] with exact solution [17] and initial condition.

$$u(x,t) = \frac{1}{2}(1+\alpha) + \frac{1}{2}(1-\alpha)\tanh\left(\sqrt{2}(1-\alpha)\frac{x}{4} + \left(\frac{1-\alpha^2}{4}\right)t\right);$$

$$u(x,0) = \frac{1}{2}(1+\alpha) + \frac{1}{2}(1-\alpha)\tanh\left(\sqrt{2}(1-\alpha)\frac{x}{4}\right)$$

Boundary conditions are extracted from the exact solution for the numerical treatment of the above FN problem at a = −22, b = 22 *and* α =0.2 at various time intervals, and the results are compared with three different RBFs naming CMRBF, MQ RBF and Gaussian RBF. TheL_∞, L_2 and RMS errors are calculated in Table 4 for different time intervals with their computational time at Δt=0.0001 and N= 41. By the analysis of derived error norms, MQ RBF is the best optimizer function as it gives accurate errors with optimized parameter value 0.209792. The graphical representation of the numerical approximations is presented by figure 2.

Table 4. Analysis of errors based on different RBFs with N=41, Iteration=10, dt=0.00001

T	**RBF**	L_∞	L_2	L_{RMS}
0.001	CMRBF	3.9815e-04	6.2180e-05	9.7109e-06
	MQ RBF	3.9768e-04	6.2108e-05	9.6996e-06
	GA RBF	-	1.7188e-04	2.6844e-05
0.002	CMRBF	8.2293e-04	1.2852e-04	2.0072e-05

	MQ RBF	7.9646e-04	1.2439e-04	1.9426e-05
	GA RBF	-	2.6347e-04	4.1148e-05
0.003	CMRBF	-	1.8637e-04	2.9106e-05
	MQ RBF	-	1.8622e-04	2.9083e-05
	GA RBF	-	5.1115e-04	7.9828e-05

Table 5 Parameter values with their computational time respect to different time at N=41, =0.00001

T	**RBF**	**Shape Parameter**	**Elapsed Time (sec)**
0.001	CMRBF	0.560914	0.013837
	MQ RBF	0.525269	0.012200
	GA RBF	0.939398	0.014185
0.002	CMRBF	0.757954	0.013277
	MQ RBF	0.881133	0.013886
	GA RBF	0.771980	0.012056
0.003	CMRBF	0.496707	0.013217
	MQ RBF	0.209792	0.015905
	GA RBF	0.935661	0.017604

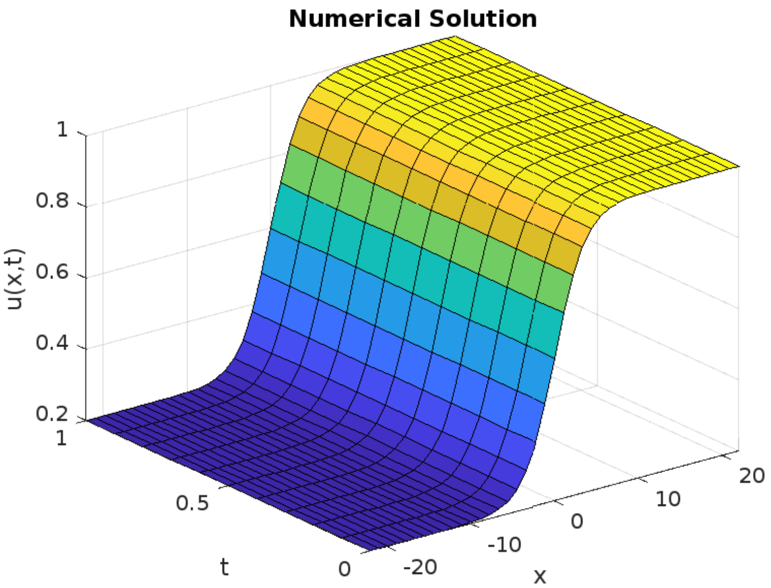

Figure 2. Numerical solution of problem 2 at N=41

Problem 3: The non-linear general Fitzhugh-Nagumo equation (1) of dimension one in the domain [-10, 10] with exact solution [17]

$$u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{1}{2\sqrt{2}}\left(x - \frac{2\zeta - 1}{\sqrt{2}}t\right)\right); \qquad x\epsilon[-10, 10] \& \ t \geq 0$$

With boundary conditions taken from the exact solution and the following initial conditions:

$$u(x,0) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{2\sqrt{2}}\right);$$

A numerical solution of the general FN equation is obtained at a = −10, b = 10 $and\ \alpha$ =0.75 at various time intervals. TheL_∞, L_2 and RMS errors are calculated in Table 6 for time intervals 0.2; 0.5; 1; 1.5 at Δt=0.0001 and N= 51. And table 7 represents the above error norms at Δt=0.001 and N=51 with same number of iterations. Results in both tables are derived by using CMRBF with their elapsed time for optimizing the RBF parameter by the analysis of error norms at different Δt. Figure 3 presents the numerical simulation of solution of FN equation at the domain points with N= 51.

Table 6 Comparison of obtained results for N=51, Δt=0.0001 and $\boldsymbol{\alpha}$ = 0.75

T **Errors**	**0.2**	**0.5**	**1.0**	**1.5**
$\boldsymbol{L_{rms}}$	7.0935e-07	1.5751e-05	1.4411e-06	5.3438e-06
$\boldsymbol{L_\infty}$	3.6177e-05	8.0332e-04	7.3496e-05	2.7253e-04
$\boldsymbol{L_2}$	5.0658e-06	1.1249e-04	1.0291e-05	3.8162e-05
CPU Time (sec.)	0.013692	0.032547	0.039668	0.059708
Shape Parameter	0.246585	0.443493	0.355015	0.001251

Table 7 Comparison of obtained results for N=51, Δt=0.001 and $\boldsymbol{\alpha}$ = 0.75

T **Errors**	**0.2**	**0.5**	**1.0**	**1.5**
$\boldsymbol{L_{rms}}$	2.2534e-07	1.1557e-05	1.0544e-04	7.0918e-05
$\boldsymbol{L_2}$	1.6093e-06	8.2532e-05	7.5299e-04	5.0646e-04
$\boldsymbol{L_\infty}$	1.1492e-05	5.8940e-04	3.8403e-04	-
CPU time (sec.)	0.015091	0.014104	0.016193	0.017701
Shape Parameter	0.046426	0.316231	0.548491	0.463863

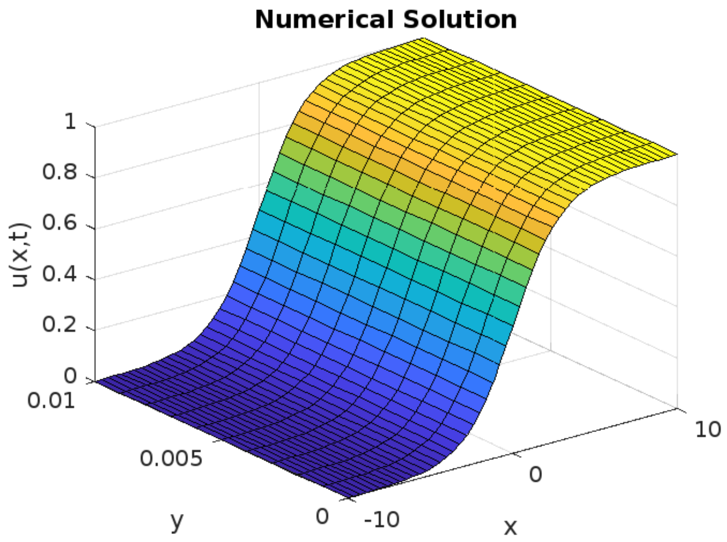

Figure 3. Graphical Representation of numerical solution of problem 3

4. Conclusion

The present article focus on a invented approach to evaluate the numerical solutions of the Fitzhugh-Nagumo (FN) equation using Artificial Bee Colony (ABC) integrated with RBF-PS method. The present approach is the hybrid form of optimization techniques and mesh free RBF methods. It has been useful in determining the parameters required to solve various mathematical models in science and engineering. The obtained numerical solutions for the three considered examples highlight the efficacy of this method and using the Gaussian RBF, Multi- quadric RBF and Cubic matern RBF are considered as basis functions for numerical approximations. The results are obtained in the form of error norms and are in good agreement with MQ RBF as presented in form of figures and tables.

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Funding and/or Conflicts of interests/Competing interests

The submitted work should be original and should not have been published elsewhere in any form or language (partially or in full). The authors declare they have no financial interests and there is no conflict of interest by all authors.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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